Prof. Dr. Oliver Landmann WS 2004/05

Final Exam "Advanced Macroeconomics"

February 28, 2005

Important information

1. Total disposable time: 90 minutes.
2. Answer all 3 questions.
3. In allocating your limited time, note the weights of the three questions as indicated.
4. Read the questions and instructions carefully. Make sure to explain the reasoning by which you reach your conclusions.

Good luck!

Question 1 (30%)

Consider an economy in the steady state of the Solow growth model. The savings rate s, the rate of depreciation δ, population growth n, the rate of technical progress g, and the production elasticity of capital α are all constant. Firms accumulate real capital up to the point where the user cost of capital is just covered by the marginal productivity of capital. The user cost of capital is the sum of real interest and depreciation. The aggregate production function is

$Y = K^{\alpha}(AL)^{1-\alpha}$ Y: Output; K: Capital; L: Labor; A: Technology

a) Explain the roles of the investment behavior of firms and the assumed behavior of savers for the steady-state capital-labor ratio and the steady-state rate of real interest in this model.

b) Determine the steady-state rate of real interest as a function of s, δ, n, g, and α.

c) In view of your result, discuss the following statement: "An economy without growth does not have a positive real interest rate."

Question 2 (40%)

The output gap of an economy *x* depends on the real interest rate *r* in the following way:

(1) $x_t = -\alpha(r_t - r^*) + \varepsilon_t^d$ ε_t^d : Stochastic demand shock with an expected value $E_{t-1}\varepsilon_t^d = 0$

The central bank controls inflation by means of an interest rate rule, setting the real interest rate *r* conditional on expected inflation $E_{t-1}\pi_t$:

(2) $r_t = r^* + \beta E_{t-1}\pi_t$

The current inflation rate π_t is given by the Philipps curve

(3) $\pi_t = \gamma x_t + E_{t-1}\pi_t + \varepsilon_t^s$ ε_t^s : Stochastic supply shock with an expected value $E_{t-1}\varepsilon_t^s = 0$

a) Give one example each of a supply shock and a demand shock.
b) What is the meaning of *r**, and which factors are reflected in this variable?
c) Using the assumption of rational expectations, determine the expectations $E_{t-1}\pi_t$ and $E_{t-1}x_t$.
d) Demonstrate how supply and demand shocks affect the output gap *x* and the inflation rate π in this model. Discuss your results.

Question 3 (30%)

How does an exogenous increase of government spending on national defence affect aggregate output, private consumption and investment in a “Real Business Cycle” model?
Explain the microeconomic logic of the effects and discuss what difference it makes whether the exogenous change is permanent or temporary.

(Hint: Additional spending on national defence increases the fraction of national income that cannot be spent on other uses.)

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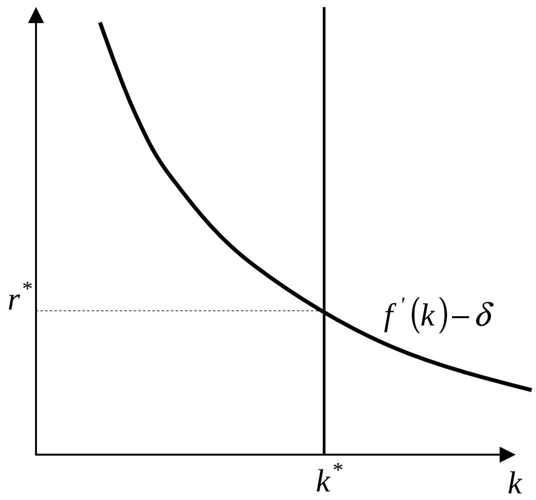
Outline of solution

Question 1

a)



The steady-state capital-labor ratio k* is uniquely determined by *s*, δ, *n*, *g*, and α as demonstrated by the Solow model. Firms' optimality condition r + δ = f'(k) implies that the steady-state real interest rate must satisfy r* = f'(k) - δ. This is the interest rate at which the supply of capital by savers and the demand for capital by firms are in balance.

b) From the steady-state condition $s\,f(k) = s\,k^{\alpha} = (n+g+\partial)k$, we get the steady-state capital-labor ration $k^* = \left[\frac{s}{n+g+\partial}\right]^{\frac{1}{1-\alpha}}$. Substituting this into the interest rate equation $r^* = f'(k^*) - \partial = \alpha k^{*\alpha-1} - \delta$ yields $r^* = \frac{\alpha}{s}(n+g+\partial) - \delta$.

c) With no growth (*n* + *g* = 0), the steady-state rate of interest is given by

r* = $\delta\left(\frac{\alpha}{s} - 1\right)$, which is positive as long as the savings rate is below the golden rule level, i.e. if s < α. (This condition is always satisfied in the context of optimal growth theory).

Question 2

a) supply shock: e.g. oil price shock
demand shock: e.g. change in consumers' or investors' sentiment.

b) *r** is the natural rate of interest, basically determined very much as spelled out in Question 1. It is the rate which balances (interest-sensitive) aggregate demand with the productive capacity of the economy. It thus reflects all the exogenous factors relevant to the determination of potential output and aggregate demand.

c) Solving (1) and (2) for x_t:

(i) $x_t = -\alpha\beta E_{t-1}\pi_t + \varepsilon_t^d$

Substituting this into (3):

(ii) $\pi_t = (1-\alpha\beta\gamma)E_{t-1}\pi_t + \gamma\varepsilon_t^d + \varepsilon_t^s$

Taking expectations:

(iii) $E_{t-1}\pi_t = (1-\alpha\beta\gamma)E_{t-1}\pi_t = 0$

Taking expectations of (i), making use of (iii):

(iv) $E_{t-1}x_t = 0$

d) Substituting (iii) into (i) and (ii):

$$x_t = \varepsilon_t^d$$

$$\pi_t = \gamma\varepsilon_t^d + \varepsilon_t^s$$

As the central bank sets the interest rate conditional on $E_{t-1}\pi_t$, it cannot respond to current supply and demand shocks. A positive demand shock raises x and π as the economy moves along its short-run Phillips curve. Supply shocks do not affect the output gap as the central bank cannot raise the interest rate in response to them.

ε_t^s is thus fully reflected in the current inflation rate.

Question 3

An exogenous increase in government spending ΔG channels resources away from private sector use and thereby exerts a negative income effect on the private sector.

If the change is *temporary*, utility maximizing households respond by reducing their leisure (thus increasing output Y) and their consumption C. This results in 0 < ΔY < ΔG. As the consumption sacrifice is spread over time, the contemporaneous fall in consumption ΔC is less than (ΔY - ΔG). Therefore, investment must fall. In sum: Y↑ = C↓ + I↓ + G↑. The resource cost of the temporary spending increase is thereby spread over present as well as future consumption and leisure.

If the same ΔG is *permanent*, consumption and leisure will immediately adjust by the full amount of the income effect. Since there is no incentive to shift the burden of adjustment between periods in this case, we get ΔY ≈ ΔC + ΔG (where ΔY > 0, ΔC < 0, ΔG > 0) with no change in investment (or a slight change depending on the capital needs of the defence industry).

(For more intuition, see Plossers article!)