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ADVANCED MACROECONOMICS I

I. Short Questions (2 points each)

Mark the following statements as True (T) or False (F) and give a brief explanation of your answer in each case.

1.	In the Ramsey-Cass-Koopmans model, the long-run equilibrium yields the highest possible level of per-capita consumption.
2.	Piketty's theory of long-run capital accumulation is inconsistent with Neoclassical steady state growth.
3.	Key feature of RBC models are optimizing households who determine the pace of technological progress endogenously.
4.	Paul Krugman's model of the baby-sitting coop explains the stickiness of wages and prices.
5.	A central bank operating under discretion reoptimizes each period.

Note: Write your answers on the blank paper provided to you.

II. 3 Problems (30 Points)

Problem 1 (8 Points)

Consider the Ramsey-Cass-Koopmans model with zero depreciation ($\delta = 0$) and a flat tax rate on capital income, τ, reimbursed via lump-sum transers T_t. Technology A_t and population N_t grow at exogenous rates g and n, respectively. Labor is supplied inelastically. Dynamics are characterized by the following equations:

(1) $$\frac{\dot{c}_t}{c_t} = \frac{(1-\tau)f'(k_t) - \rho - \theta g}{\theta}$$

(2) $$\dot{k}_t = f(k_t) - c_t - (n+g)k_t$$

subject to a transversality condition.

(a) Explain equations (1) and (2). Why does T_t not show up anywhere?

(b) How does the capital income tax τ affect the steady state values of c and k?

(c) *"A cut in capital income taxes raises the after tax rate of return and hurts wage income."* Discuss.

Problem 2 (10 Points)

Consider the following model of inflation and output gaps:

(1)	$\dot{\pi}(t) = \alpha y(t)$	Inflation dynamics
(2)	$y(t) = -\sigma^{-1} r(t)$	Output determination
(3)	$r^T = \phi[\pi(t) - \pi^T]$	Real interest rate target
(4)	$\dot{r}(t) = \lambda[r^T - r(t)]$	Real interest rate adjustment

where $\pi(t)$ is the inflation rate, $y(t)$ is the deviation of output from potential output, and $r(t)$ the real interest rate. π^T is the central bank's inflation target. α, σ, λ and ϕ are positive structural parameters.

(a) Explain the theoretical motivation underlying equations (1)-(4).

(b) Express the dynamics of the output gap, $\dot{y}_t$ as a function of y_t and π_t. Sketch the dynamics of output and inflation in (y,π) space.

(c) According to the above model, how does the central bank adjust the *nominal* interest rate in response to a change in the inflation rate? Explain.

Problem 3 (12 Points)

Consider the following Lucas model of an economy:

(1) $$y_t = b\,(p_t - E_{t-1}[p_t])$$

(2) $$y_t = m_t - p_t + \varepsilon_t^d$$

(3) $$m_t = \bar{m} - cy_t$$

where y denotes the log of output, m the log of money supply and p is the log of the price level. E_t is the expectations operator conditioning on all information available at t. The disturbance ε^d is white noise and independently distributed with expected values of zero and variance V_d. The parameters b and c are strictly positive.

(a) Whose behavior is described by equations (1)-(3)? Explain what the three equations say about the determination of output and the money supply. What is ε_t^d? Give an example.

(b) Solve the system (1)-(3) for p_t as a function of $E_{t-1}p_t$, ε_t^d and $\bar{m}$.

(c) Determine the rational expectation $E_{t-1}p_t$.

(d) Determine p_t, assuming rational expectations.

(e) How does the choice of the parameter c affect the stability of the price level?

SOLUTION

Problem	Points					Total
I	1P T/F, 1P expl.					10
	(a)	(b)	(c)	(d)	(e)	
II1	3	2.5	2.5	-	-	8
II2	4	4	2	-	-	10
II3	4	3	2	1	2	12

I. Short Questions (10 Points)

1. **False.** Since households' discount rate is assumed to exceed the growth of effective labor input, steady-state consumption per effective worker is lower than the highest possible (golden-rule) level.
2. **True.** According to Piketty, capital grows more rapidly than output because of $r > g$ and the reinvestment of the entire return on capital. In contrast, neoclassical growth theory predicts that capital grows at the same rate as output in the steady state.
3. **False.** The pace of technological progress follows an exogenous process.
4. **False.** It explains how stickiness of wages and prices affect real allocations.
5. **True.** The central bank cannot commit to future actions and therefore picks values of the nominal interest rates to influence output and inflation, taking as given all future outcomes.

II. 3 Problems (30 Points)

1. (a) Equ. (1) is the consumption Euler equation and follows from households' optimization. Households choose their consumption path with regard to the (net) rate of return on capital and their time preference. The sensitivity of the slope of the consumption path reflects the intertemporal elasticity of substitution θ. [1P]

   Equ. (2) is the law of motion for capital derived from the resource constraint of the economy. Additional capital per-effective worker k is going to be accumulated whenever output is not entirely used up by consumption and by the break-even investment that is required to maintain k. [1P]

   T_t does not show up since it neither affects incentives for intertemporal substitution (1) nor the resources available to the economy in the aggregate because $(1-\tau)f'(k_t) = T_t$. [1P]

   — [3P]

   (b) From (1), the steady-state condition $\dot{c} = 0$ implies

$$f'(k) = \frac{\rho + \theta g}{1 - \tau}.$$

   For higher values of τ, the RHS increases. This is consistent with lower values of k in the steady state because $f'(\cdot)$ is a decreasing function. According to (2), c decreases as well because $f(k) - (n+g)k$ is increasing in k on the relevant range. This can be seen from the phase diagram, where the isocline for $\dot{c} = 0$ is shifted to the left. This logic applies globally, since $\tau \in [0, 1]$. [2.5P]

   (c) The steady-state after-tax rate of return on capital equals $\rho + \theta g$ and does not depend on τ, therefore. A cut in capital income taxes increases the steady-state amount of capital in the economy and hence the marginal product of labor (under complementarity of factors of production). Thus, the wage rate is improved by a cut in the capital income tax. In the short run, i.e. before the capital stock has had time to adjust to the tax cut, the effects are quite different: The tax cut increases the net return on capital, leaves the wage rate unchanged and reduces tax revenue in the short run. Since workers share in the lump-sum rebate, they actually suffer in the short term. [2.5P]

2. (a) Equ. (1) states that the change of the inflation rate depends on the output gap, as implied by an accelerationist Phillips curve. [1P]

   Equ. (2) stipulates that the output gap is a function of the real interest-rate gap (the difference between the real interest rate and the natural interest rate). This follows from the IS function and the definition of the natural interest rate. [1P]

   Equ. (3) relates the real interest target to the inflation target of the central bank. By setting the interest rate above the natural rate whenever inflation exceeds the target rate, the central bank counteracts any inflationary pressure. [1P]

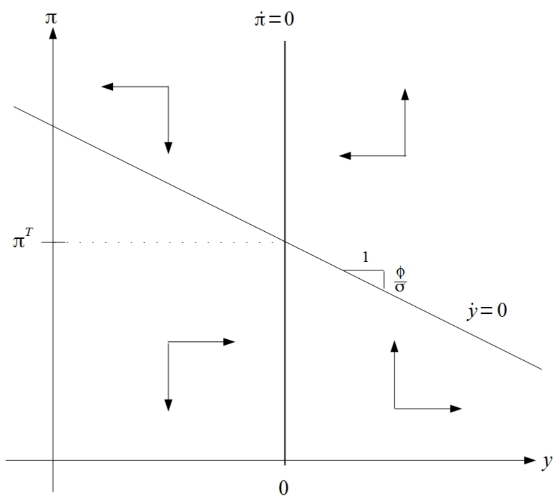
   Equ. (4) expresses interest-rate smoothing on the part of the central bank, by which the interest rate is adjusted towards the target rate only gradually (lest financial markets get unsettled). [1P]

   — [4P]

   (b) With slight abuse of notation: write x_t for $x(t)$. Combining equations (2)-(4), it can be shown that

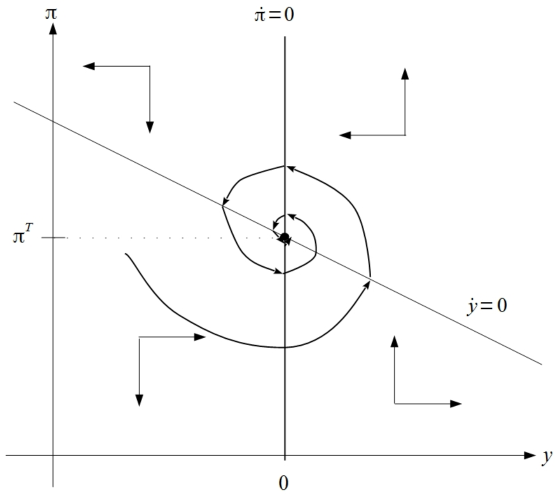
$$\begin{aligned} \dot{y}_t &= \frac{\partial y_t}{\partial t} = -\frac{1}{\sigma}\frac{\partial r_t}{\partial t} = -\frac{\dot{r}_t}{\sigma} = -\frac{\lambda(r^T - r_t)}{\sigma} \\ &= -\frac{\lambda}{\sigma}\left[\phi(\pi_t - \pi^T) + \sigma y_t\right] \\ \Leftrightarrow \quad \dot{y}_t &= -\frac{\lambda\phi}{\sigma}(\pi_t - \pi^T) - \lambda y_t. \end{aligned} \quad (2P)$$

   Together with equation (1), the corresponding (y, π) phase diagram looks as follows:



   Inflation stays constant for $y_t = 0$, rises for $y_t > 0$, and falls for $y_t < 0$.

Positive output deviations get larger [smaller] when output and inflation are below [above] the $\dot{y} = 0$-locus (the region where the central bank will lower [increase] real interest rates). Output and inflation move counter-clockwise around the long-run equilibrium $(0, \pi^T)$:



Note: One graph sufficient. [2P]

— [4P]

(c) The central bank will engineer a more than one-by-one increase in nominal interest rates in response to changes to inflation rates because $\phi > 0$. [1P]
Thus, the central bank obeys the Taylor principle. This is necessary to have a stable long-run equilibrium, i.e. contracting circular movements around $(0, \pi^T)$. [1P]

— [2P]

3. (a) Equ. (1) is the Lucas supply curve, describing the output supplied by producers. The coefficient b expresses the price elasticity of supply which reflects the signal-extraction problem of firms who must infer their relative prices form observed absolute prices. [1P]

Equ. (2) describes aggregate demand as a function of agents' real balances $m_t - p_t$. ε_t^d expresses an exogenous demand shock (such as a change in agents' spending behavior or an increase in government spending). [1P]

Equ. (3) describes how the central bank sets monetary policy. In an attempt to stabilize the economy, it reduces the money supply below $\bar{m}$ whenever output is above equilibrium (and vice versa in the opposite case). [1P]

Output is determined by the market-clearing equilibrium of supply and demand. Since the central bank determines the supply of money, it indirectly affects output as well. [1P]

— [4P]

(b) Combining (2) and (3) yields:

$$
\begin{aligned}
y_t &= \bar{m} - cy_t - p_t + \varepsilon_t^d \\
\Leftrightarrow\ y_t &= \frac{1}{1+c}\left(\bar{m} - p_t + \varepsilon_t^d\right) \qquad \text{(1P)}
\end{aligned}
$$

Now setting that equal to (1) implies:

$$
\begin{aligned}
b(p_t - E_{t-1}p_t) &= \frac{1}{1+c}\left(\bar{m} - p_t + \varepsilon_t^d\right) \\
\Leftrightarrow\ p_t\left(b + \frac{1}{1+c}\right) &= \frac{1}{1+c}\left(\bar{m} + \varepsilon_t^d\right) + bE_{t-1}p_t \\
\Leftrightarrow\ p_t\left(\frac{b(1+c)+1}{1+c}\right) &= \frac{1}{1+c}\left(\bar{m} + \varepsilon_t^d\right) + bE_{t-1}p_t \\
\Leftrightarrow\ p_t &= \frac{1}{1+b(1+c)}\left(\bar{m} + \varepsilon_t^d + (1+c)bE_{t-1}p_t\right) \qquad \text{(2P)}
\end{aligned}
$$

(c) Taking rational expectations of p_t and solving for $E_{t-1}p_t$:

$$
\begin{aligned}
E_{t-1}p_t &= E_{t-1}\left[\frac{1}{1+b(1+c)}\left(\bar{m} + \varepsilon_t^d + (1+c)bE_{t-1}p_t\right)\right] \\
\Leftrightarrow\ E_{t-1}p_t &= \frac{1}{1+b(1+c)}\left(\bar{m} + (1+c)bE_{t-1}p_t\right) \\
\Leftrightarrow\ [1+b(1+c)]E_{t-1}p_t &= \bar{m} + (1+c)bE_{t-1}p_t \\
\Leftrightarrow\ E_{t-1}p_t &= \bar{m}
\end{aligned}
$$

(2P)

(d) Inserting the previous result into p_t, one obtains

$$\begin{aligned} p_t &= \frac{1}{1+b(1+c)}\left(\bar{m}+\varepsilon_t^d+(1+c)b\bar{m}\right) \\ &= \frac{1}{1+b(1+c)}\left(\bar{m}[1+b(1+c)]+\varepsilon_t^d\right) \\ &= \bar{m}+\frac{1}{1+b(1+c)}\varepsilon_t^d \end{aligned} \qquad \text{(1P)}$$

(e) The variance of the price level is given by:

$$V(p_t) = V\left(\bar{m}+\frac{1}{1+b(1+c)}\varepsilon_t^d\right) = \frac{1}{[1+b(1+c)]^2}V_d \qquad \text{(1P)}$$

because $\bar{m}$ is a constant/parameter and ε_t^d has constant variance V_d. Hence, the stability of the price level *increases* for larger values of c because demand shocks are mitigated. [1P]

— [2P]