ADVANCED MACROECONOMICS I

Final Exam, Winter Semester 2018/19, February 13, 2019

I. Short Questions (2 points each)

Explain for each of the following statements why you agree or disagree.

1.	The assumptions of the Solow growth model about production technology and market structure imply constant income shares for labor and capital in the steady state.
2.	The famous "Elephant Chart" presented by Branko Milanovic is consistent with the observation that income inequality has decreased globally while increasing within countries.
3.	According to the Lucas model, the long-run effect of a monetary policy shock on output depends on whether or not the shock was unanticipated in the short run.
4.	According to the Real Business Cycle model, the long-run effect of a monetary policy shock on output depends on the serial correlation of the shock.
5.	In the New Keynesian model presented by Clarida, Galí and Gertler, the optimal policy response to a demand shock depends on the relative weights of output and inflation stabilization in the central bank's objective function.

II. 3 Problems (30 Points)

Problem 1 (10 Points)

A household who chooses a continuous time path for a control variable $c(t)$ so as to maximize

$$\int_{t=0}^{\infty} e^{-\rho t} f(x,c)dt \quad \text{s.t. } \dot{x} = g(x,c),\ x(0) = x_0$$

does so by setting up the current-value Hamiltonian

$$\mathcal{H} = f(x,c) + \mu g(x,c)$$

and satisfying the first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial c} &= 0 \\
\frac{\partial \mathcal{H}}{\partial x} &= \rho\mu - \dot{\mu} \\
\frac{\partial \mathcal{H}}{\partial \mu} &= \dot{x}
\end{aligned}$$

where $x(t)$ is a state variable with initial value x_0.
For the following analysis, assume $f(x,c) = L(t)\log c(t)$, where $L(t) = L(0)e^{nt}$ is the number of household members, and $c(t)$ is per-capita consumption. $x(t)$ is per-capita wealth of the household which evolves according to

$$\dot{x} = rx(t) + w - c(t) - nx(t)$$

where r is the rate of return on wealth and w is per-capita wage income.

(a) Explain the difference between a control and a state variable.

(b) Making use of the Hamiltonian, derive the household's optimality conditions and show what they imply for the dynamic behavior of consumption over time.

(c) Explain the role of n for optimal consumption behavior.

Problem 2 (10 Points)

Consider the following model of inflation and unemployment:

$$\dot{\pi}(t) = -\alpha[u(t) - u_n] \tag{1}$$
$$u(t) - u_n = \sigma[r(t) - r_n] \tag{2}$$
$$r^{TR}(t) = r_n + \gamma[\pi(t) - \pi^T] - \varepsilon[u(t) - u_n] \tag{3}$$
$$\dot{r}(t) = \lambda[r^{TR}(t) - r(t)] \tag{4}$$

where $\pi(t)$ is the inflation rate, $u(t)$ is the unemployment rate, and $r(t)$ the real interest rate. π^T is the central bank's inflation target. α, σ, γ, ε and λ are positive structural parameters.

(a) What are the theoretical underpinnings of equations (1) and (2)?

(b) What do equations (3) and (4) tell you about how the interest rate is determined? What does r^{TR} stand for and what is the interpretation of the terms on the RHS of equation (3)?

(c) "If the nominal interest rate hits the zero lower bound, the dynamics of inflation and unemployment are fundamentally changed and a deflationary spiral is the inevitable consequence."
Do you agree or disagree? Provide an explanation in the context of the presented model.

Problem 3 (10 Points)

(a) What is the Solow residual and how is it related to the concept of technology shocks in the Real Business Cycle theory?

(b) What role does Real Business Cycle theory attribute to the Solow Residual in its account of short-run output fluctuations?

(c) Describe the income and substitution effects acting on labor supply when the economy is hit by a positive technology shock.

SOLUTION

Problem	Points			Total
I	2P each			10
	(a)	(b)	(c)	
II1	3	5	2	10
II2	4	3	3	10
II3	3	3	4	10

I. Short Questions (10 Points)

1. The Solow growth model assumes constant returns to scale and perfect competition. Therefore, the factor prices equal their marginal productivity. Since marginal and average productivities are constant in the steady state, so are factor shares.

2. The elephant chart shows significant gains for global top income recipients, "The Global Plutocrats", mostly in the richest countries (trunk of elephant). Thus, inequality within rich countries has increased. At the same time, a global middle class emerged, mostly in Asia (body of elephant), which is catching up with average incomes of the rich countries while pulling away from their poorer compatriots. As a result, inequality has indeed decreased globally while increasing within countries.

3. No, in the long run of the Lucas model, money is neutral regardless of whether or not there have been surprises in the short run.

4. No, the statement cannot be correct because in the RBC model, money does not play any role at all.

5. No, a demand shock drives the output and inflation gap in the same direction (both positive or both negative). Thus, the central bank does not face a trade-off between output and inflation stabilization and relative weights in the objective function do not matter.

II. 3 Problems (30 Points)

1. (a) A control variable can be adjusted directly and without lag to a new (supposedly optimal) value, like consumption $c(t)$ in this example. In contrast, state variables are given in the beginning of a period and can at most be determined indirectly, like assets $x(t)$ here.

   (b) For the presented functional forms the Hamiltonian is given by:

$$\mathcal{H} = L(t) \log c(t) + \mu[rx(t) + w - c(t) - nx(t)]$$

   associated with the first-order conditions

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{L(t)}{c(t)} - \mu = 0, \tag{1}$$

$$\frac{\partial \mathcal{H}}{\partial x} = \mu(r - n) = \rho\mu - \dot{\mu}, \tag{2}$$

$$\frac{\partial \mathcal{H}}{\partial \mu} = rx(t) + w - c(t) - nx(t) = 0. \tag{3}$$

   To obtain $\dot{\mu}$, derive the FOC (1) w.r.t. time as to obtain

$$\frac{nL(0)e^{nt}c(t) - L(t)\dot{c}(t)}{c(t)^2} = \dot{\mu}.$$

   Simplifying the FOC (2) by division of μ:

$$r - n = \rho - \frac{\dot{\mu}}{\mu}.$$

   Taking these two equations together and simplifying:

$$r - \rho - n = -\frac{[nL(t)c(t) - L(t)\dot{c}(t)]}{c(t)^2}\frac{c(t)}{L(t)} = \frac{\dot{c}(t)}{c(t)} - n,$$

   so that the dynamics of consumption are given by the Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = r - \rho.$$

   (c) Population growth makes it more costly to increase household wealth, since aggregate wealth is distributed between more household members (LHS of (2), reflecting effect on budget constraint). On the other hand, population growth makes it more valuable to invest into assets as to maintain a constant value (marginal utility) from consumption, since additional household members also raise the utility integral (RHS of (2), reflecting growth in the functional $f(x, c)$). The two effects are balanced, so that n cancels out in the Euler-equation.

2. (a) Equation (1) is the Accelerationist Phillips curve. The accelerationist property results from the inclusion of inflation expectations in the Phillips Curve in combination with the assumption of an adaptive (backward-looking) adjustment of expectations.
Equation (2) reflects the joint hypothesis of an IS-type interest rate-demand relationship and Okun's law: an increase in the interest rate reduces economic activity (e.g. via diminished investment or consumption), which drives up unemployment.

   (b) Equation (3) describes a Taylor rule benchmark for the real interest rate according to which the central bank adjusts the nominal interest so as to raise the real interest rate whenever inflation moves above the target rate (cooling down inflationary pressure) or unemployment falls below the natural equilibrium rate (fighting an incipient recession). Equation (4) says that the actual real interest rate $r(t)$ is adjusted towards the Taylor rate only gradually ("interest-rate smoothing").

   (c) The dynamics are not necessarily "fundamentally" changed. The ZLB only bites when the central bank would like to lower nominal interest rates as to decrease real interest rates, thereby stimulating demand. According to the model (1)-(4), a deflationary spiral only ensues whenever the ZLB bites at $u > u_n$: there the binding ZLB implies constant nominal interest rates at accelerating deflation. Looking at the Fisher-relation, this implies rising real interest rates. For $u < n_n$, the Phillips curve implies accelerating inflation. The economy experiences a protracted recovery along which real interest rates decrease slowly (Fisher equation) since the central bank is stuck at $i = 0$.
Hence, a deflationary spiral is a possible, but not an inevitable consequence of a binding ZLB.

3. (a) The rate of technological progress (total factor productivity growth) is hard to measure directly. The growth rate of output and the production factors are measurable, however. Therefore, the rate of technological progress is determined as the residual of output growth that cannot be attributed to the growth of the production factors capital and labor, weighted with their respective partial elasticities. The fluctuations in this so called Solow residual around the long-time trend are then taken to be exogenous fluctuations in technology by the RBC model.

   (b) In the RBC model, technology shocks are considered to be a major source of business fluctuations. The induced fluctuations in output, investment, consumption and employment are then interpreted as optimal responses of the private sector to those shocks.

   (c) The marginal product of labor (MPL = real wage) rises when the economy is hit by a positive productivity shock. This has three direct effects on the labor supply:

      (1) An intratemporal substitution effect potentially raises labor supply, because leisure has become more expensive in terms of consumption goods.

      (2) An intertemporal substitution effect pronounces the positive incentive to work because the higher wage is interpreted as transitory due to the assumed AR(1) structure of technology. Labor input is shifted form the future to the current high-wage period (Intuitively, "make hay while the sun shines").

      (3) An income effect: households get richer and can afford more of all their goods, including leisure. This means that households would like to work less when leisure is a normal (superior) good.

      RBC theory assumes that substitution effects dominate the income effect for transitory shocks.