ADVANCED MACROECONOMICS I

Final Exam, Winter Semester 19/20, February 26, 2020

I. Short Questions (2 points each)

Explain for each of the following statements why you agree or disagree.

1.	According to Kaldor's stylized growth facts, productivity growth is equalized across countries.
2.	In the Ramsey-Cass-Koopmans model, a time preference rate above the steady-state rate of growth of output ensures dynamic efficiency.
3.	The parable of the baby-sitting economy supports the idea that monetary policy is effective in addressing coordination failures in a market economy.
4.	According to Calvo's model of staggered prices, all firms operate under monopolistic competition and reset prices in each period.
5.	In the New Keynesian model presented by Clarida, Galí and Gertler, the location of the Phillips curve reflects the central bank's optimal response to cost-push shocks.

II. 3 Problems (30 Points)

Problem 1 (6 Points)

Consider a growth model where production obeys the Cobb-Douglas production function $Y = (A_t L_t)^{1-\alpha}(T \cdot K_t)^{\alpha}$, where T is use of land. Population L_t and technology A_t grow at their respective exogenous rates n and g. The capital stock evolves according to $\dot{K} = Y - C - \delta K$, where $C = (1-s)Y$. T is constant.

a) Derive the law of motion for capital per-effective worker $k \equiv \frac{K}{AL}$.

b) Is there a steady state in k?

c) Does your answer to b) change, if T grows at rate g_T?

Problem 2 (12 Points)

Consider a model consisting of the following equations:

$$y_t = \bar{y} + b\,(p_t - \mathbb{E}_{t-1}[p_t]) + \varepsilon_t^s \tag{1}$$
$$y_t = \bar{d} + m_t - p_t \tag{2}$$
$$m_t = \bar{m} - c\,(y_{t-1} - \bar{y}) \tag{3}$$

where y denotes output, p the price level and m the money supply. $\mathbb{E}_s$ is the expectations operator conditioning on all information available at s. The disturbance ε^s has an expected value of zero and variance V_s. The parameters b and c are strictly positive.

a) What is the microeconomic foundation underlying eq. (1)? Explain briefly and without resorting to a formal model.

b) Assume the price level surprise is

$$p_t - \mathbb{E}_{t-1}p_t = -\frac{1}{1+b}\varepsilon_t^s. \tag{4}$$

i. Use this assumption to solve for p_t and y_t in the model (1)-(3).

ii. Verify for your solution that the initial assumption of eq. (4) is consistent with rational expectations.

c) How does the choice of the coefficient c in eq. (3) affect the volatility of y and p, given the variance of ε_t^s?

Problem 3 (12 Points)

Consider the model consisting of the following equations:

$$\dot{\pi}(t) = -\alpha\,[u(t) - u_n] \qquad \text{Accelerationist Phillips curve} \qquad (1)$$

$$y(t) = D - \delta r(t) \qquad \text{Aggregate Demand (IS-curve)} \qquad (2)$$

$$u(t) - u_n = -\beta\,[y(t) - \bar{y}] \qquad \text{Okun's Law} \qquad (3)$$

$$r^{TR}(t) - \bar{r} = \gamma[\pi(t) - \pi^T] - \varepsilon[u(t) - u_n] \qquad \text{Taylor rule} \qquad (4)$$

$$\dot{r}(t) = \lambda[r^{TR}(t) - r(t)] \qquad \text{Interest rate smoothing} \qquad (5)$$

where $\pi(t)$ is the inflation rate, u the rate of unemployment, y the log of output, and r the real interest rate. The natural rate of unemployment u_n, the log of autonomous demand D and the central bank's targets π^T and $\bar{r}$ are exogenously determined. The parameters α, β, δ, γ, ε and λ are strictly positive.

a) Compute the long-run equilibrium values of the endogeneous variables.

b) Does the central bank achieve its inflation target in the long-run?

c) Suppose $\bar{r} = r_n$ and there is a lower bound (ZLB) on nominal interest rates.

   i. What determines the likelihood of the ZLB being hit in the event of an exogenous shock?

   ii. Does the economy inevitably end up in a downward deflationary spiral if the central bank hits the ZLB?

SOLUTION

Problem	Points			Total
I	2P each			10
	(a)	(b)	(c)	
II1	2	3	1	6
II2	3	6	3	12
II3	5	2	5	12

I. Short Questions (10 Points)

1. False. Kaldor's stylized facts state that productivity can differ considerably across countries.

2. Right. When the rate of return to capital exceeds the growth rate of output ($n + g$), the economy will not over-accumulate capital if

$$r_t = f'(k) = \rho + \theta g > n + g$$

$\rho > n + g$ is sufficient for this inequality to hold strictly.

3. Right. Monetary policy can be helpful in addressing the issue of price stickiness as an example of a coordination failure. Policy options are not restricted to monetary policy to remedy with or establish the coordination mechanism of pricing.

4. False. A constant fraction of firms cannot reset prices in each period.

5. Right. Inflation expectations determine the location of the New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda x_t + u_t$$

in the (π_t, x_t) space. Optimal monetary policy implies stabilization of inflation in each period. This in turn reduces inflation expectations, since inflation is lower given the series of shocks.

II. 3 Problems (30 Points)

Problem 1 (6 Points)

a) The definition of capital per-effective worker implies

$$\frac{\dot{k}}{k}g_k = g_K - g_A - g_L = \frac{\dot{K}}{K} - (n+g).$$

The law of motion for $\dot{K}$ after inserting consumption implies thus

$$\begin{aligned}\dot{k} &= \frac{sY - \delta K}{K}k - (n+g)k = s\frac{Y}{K} - (n+g+\delta)k \\ &= s\frac{(AL)^{1-\alpha}(TK)^{\alpha}}{K}k - (n+g+\delta)k = sT^{\alpha}k^{\alpha} - (n+g+\delta)k\end{aligned}$$

b) The law of motion depends on T, but T here is assumed to be a constant. The steady state would be given by

$$k^* = T^{\frac{\alpha}{1-\alpha}}\left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

so that we can find a steady state in k when T is constant.

c) When T depends on time, $k = \frac{K}{AL}$ is not an appropriate normalization.

Problem 2 (12 Points)

a) Producers face a signal extraction problem: their price p_i is a noisy signal for the relevant price r_i which relates p_i to the aggregate price p. The surprise $p_t - \mathbb{E}_{t-1}[p_t]$ here captures the idea that when actual aggregate prices p_t deviate from their expectations $\mathbb{E}_{t-1}p_t$, more firms will over-produce than under-produce.

b) i. Inserting the price level surprise into eq. (1) and combining the equations (1)-(3) after reformulations yields

$$p_t = \bar{d} + \bar{m} - c(y_{t-1} - \bar{y}) - \bar{y} - \frac{1}{1+b}\varepsilon_t^s$$

Output follows immediately from (1):

$$y_t = \bar{y} - \frac{b}{1+b}\varepsilon_t^s + \varepsilon_t^s = \bar{y} + \frac{1}{1+b}\varepsilon_t^s$$

ii. Applying rational expectations to p_t:

$$\mathbb{E}_{t-1}p_t = \bar{d} + \bar{m} - c(y_{t-1} - \bar{y}) - \bar{y} - \frac{1}{1+b}\mathbb{E}_{t-1}[\varepsilon_t^s]$$

where $\mathbb{E}_{t-1}[\varepsilon_t^s] = 0$ implies that the surprise is given by

$$p_t - \mathbb{E}_{t-1} p_t = -\frac{1}{1+b}\varepsilon_t^s$$

which verifies the initial assumption.

c) Monetary policy parameter c does not enter y_t and affects variability of p_t only through its effect via y_{t-1} so that

$$V(p_t) = c^2 \underbrace{V(y_{t-1})}_{=\frac{1}{(1+b)^2}V(\varepsilon_t^s)} + \frac{1}{(1+b)^2}V(\varepsilon_t^s) = \frac{1+c^2}{(1+b)^2}V(\varepsilon_t^s)$$

which implies that $c = 0$ minimizes variability of prices.

Problem 3 (12 Points)

a)
- Constant inflation requires $u(t) = u_n$ in eq. (1).
- $u(t) = u_n$ implies $y(t) = \bar{y}$ according to eq. (3).
- $y(t) = \bar{y}$ implies $r(t) = \frac{D-\bar{y}}{\delta} \equiv r_n$ according to (2).
- $u(t) = u_n$ implies $r^{TR} = \bar{r} + \gamma(\pi(t) - \pi^T)$ by eq. (4),
- while $r(t) = r_n$ implies $\dot{r}(t) = 0$ and therefore $r^{TR} = r(t) = r_n$ by eq. (5).

Hence, the Taylor rule implies $r_n - \bar{r} = \gamma(\pi(t) - \pi^T)$ so that $\pi(t) = \pi^T + \frac{r_n - \bar{r}}{\gamma}$.

b) The central bank only achieves $\pi(t) = \pi^T$ for $t \to \infty$ if it sets $\bar{r} = r_n$. Else, inflation converges to the value described in a).

c) i. Given the parameters, the location of the long-run equilibrium and the ELB is determined by the values u_n, π^T and r_n. Increases in...
- $r_n = \frac{D-\bar{y}}{\delta}$ shift the ZLB down,
- π^T shift the long-run equilibrium up,
- u_n shift the ZLB up while shifting the long-run equilibrium to the right.

Thus, increases in π^T and r_n (via increases in D or decreases in $\bar{y}$) unambigiously decrease the likelihood of hitting the ZLB.

ii. The economy does not end up in a deflationary spiral when the ZLB is hit at "over-employment" $u_t < u_n$, which is where the economy experiences a protracted recovery.