ADVANCED MACROECONOMICS I

Retake Exam, Winter Semester 2018/19, August 26, 2019

I. Short Questions (2 points each)

Explain for each of the following statements why you agree or disagree.

1.	Paul Romer and William Nordhaus were awarded the Nobel Prize in Economics in October 2018 for work that has decisively refuted the Solow model of economic growth.
2.	The close correlation of Y/L and K/L across countries indicates that the accumulation of physical capital is the quantitatively most important determinant of living standards.
3.	β-convergence is sufficient for σ-convergence.
4.	According to the consumption Euler-equation, households follow a steep consumption path when interest rates are high.
5.	The Lucas model and Real Business Cycle Theory share the implication that systematic counter-cyclical policy cannot provide for more efficient shock absorption than the private sector does on its own.

II. 3 Problems (30 Points)

Problem 1 (10 Points)

Consider the Solow model where production $Y_t = F(AL, K)$ is subject to constant returns to scale. Technology A is determined endogenously. The rate of technological progress is g and population L grows at rate n. Capital in efficiency units is defined as

$$k = \frac{K}{AL} \tag{1}$$

A fraction s of output is devoted to capital accumulation. The capital stock depreciates at rate δ.

(a) Derive the law of motion for the capital stock in efficiency units ($\dot{k}$) from the information above.

(b) The economy is in an initial steady state when a fraction of the labor force is suddenly lost due to an epidemic. Explain how Y, Y/L and k are affected by this event in the short run and in the long run.

Problem 2 (10 Points)

Consider a variant of "the world's smallest macroeconomic model" in which a single consumption good C is produced by the single factor of production, labor L, with a given labor productivity λ:

$$C = \lambda L \tag{1}$$

Households are endowed with a fixed amount of money M and end with an amount M' after spending on consumption and earning income. They inelastically supply an amount of labor L_0. All income generated by the economy is received by households. Their utility function is

$$U = (1-s)\log C + s\log(M'/P), \tag{2}$$

where P is the price of the consumption good and $s \in (0, 1)$.

(a) Show that the consumption demand function is given by

$$C^d = (1-s)(\lambda L + M/P).$$

(b) Determine C, L and P as a function of M and λ, assuming perfectly flexible price adjustment.

(c) How does a doubling of λ affect C, L and P at given M and under perfect price flexibility?

(d) In contrast, how does a doubling of λ affect C and L if P is rigid? What is the lesson for the macroeconomic management of productivity growth in a market economy?

Problem 3 (10 Points)

Consider the following variant of the Clarida-Galí-Gertler (1999) model - in usual notation. The central bank minimizes L_t and is subject to the following system:

$$L_t = \alpha x_t^2 + (\pi_t - \pi)^2 + \delta \mathbb{E}_t L_{t+1} \tag{1}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda x_t + u_t \tag{2}$$

$$x_t = \mathbb{E}_t x_{t+1} - \varphi \left(i_t - \mathbb{E}_t \pi_{t+1} - r_n \right) + g_t \tag{3}$$

where u and g follow AR(1) processes as in the paper. α, β, λ and φ are given parameters. r_n is the real interest rate consistent with output at its natural level. $\mathbb{E}_t[\cdot]$ are the rational expectations at time t.

(a) Interpret the central bank's loss function. Why does the loss function include a term $\delta \mathbb{E}_t L_{t+1}$?

(b) What is the meaning of discretionary policy in this context, and what is implied for the optimization problem of the central bank?

(c) Consider a stationary long-run equilibrium without shocks to u and g. How does the central bank set i_t in this scenario?

(d) Sketch the adjustment path of x_t and π_t when the economy is hit by a negative cost-push shock ($u_t < 0$).

SOLUTION

Problem	Points				Total
I	1P T/F, 1P expl.				10
	(a)	(b)	(c)	(d)	
II1	4	6	-	-	10
II2	3	2	2	3	10
II3	2	2	3	3	10

I. Short Questions (10 Points)

1. **False.** Romer and Nordhaus were awarded the Nobel Prize for contributions to endogenous growth theory and climate change. These do not refute the Solow model of economic growth.

2. **False.** The correlation between Y/L and K/L can be immaterial to living standards, when causation is pointing from Y to K.

3. **False.** β-convergence is not sufficient for σ-convergence in stochastic environments.

4. **True.** The Euler-equation prescribes a higher growth rate of consumption whenever interest rates are high. Thus, the corresponding consumption path gets steeper.

5. **True.** In the Lucas model, counter-cyclical policy responses of monetary policy imply higher variances of production and prices. In the RBC model, there are no structural deviations from the first welfare theorem, so that responses of the economy cannot be improved upon.

II. 3 Problems (30 Points)

Problem 1 (10 Points)

(a) The law of motion for aggregate capital is

$$\dot{K} = sY - \delta K$$

so that the capital in efficiency units k follows

$$\dot{k} = g_k k = \frac{\partial \log k}{\partial t} k = \frac{\partial \log(K/(AL))}{\partial t} k = (g_K - g - n)k$$

Thus, inserting back $k = K/(AL)$ it follows that

$$\dot{k} = \frac{K}{AL}\frac{\dot{K}}{K} - (n+g)k = \frac{sY - \delta K}{AL} - (n+g)k = sy - (n+g+\delta)k$$

where $y = Y/(AL) = F(AL, K)/(AL) = F(1, k)$.

(b) Epidemic: L goes down. In the short run, K and A are fixed: they are state variables. Hence, $Y = F(AL, K)$ goes down while $Y/L = F(A \cdot 1, K/L)$ and $k = K/(AL)$ increase.
In the long run, the economy's properties are determined by structural parameters and equations. Hence, k returns to its steady state value, while Y and Y/L bounce back to their growth rates $n + g$ and g, respectively. On the convergence path back to the steady state in k, aggregate variables like Y and per-capita variables Y/L experience higher growth rates.

Problem 2 (10 Points)

(a) Maximizing utility expressed in (2) subject to the budget constraint

$$C + M'/P = \lambda L + M/P.$$

yields the result. Parameter s can be interpreted as the savings rate:

$$S = s\left(\lambda L + \frac{M}{P}\right)$$

Total savings S together with consumption exhaust the initial endowment:

$$C + S = \lambda L + \frac{M}{P}$$

It follows that consumption demand for the savings rate s is given by

$$C^d = (1-s)(\lambda L + M/P)$$

(b) Equilibrium conditions for the goods and labor market:

$$C^d = C^s = \lambda L_0 \text{ and } L = L_0.$$

Thus:

$$(1-s)(\lambda L_0 + M/P) = \lambda L_0.$$

Rewriting for P implies the equilibrium price

$$P = \frac{1-s}{s}\frac{M}{\lambda L_0}.$$

Equilibrium output and employment follow from the equilibrium conditions immediately: $L = L_0$ and $C = \lambda L_0$.

(c) After a doubling of productivity, we have

$$C = (1-s)(2\lambda L + M/P) \text{ and } L = \frac{C}{2\lambda},$$

where λ is original productivity. The new price level is given by

$$P' = \frac{1-s}{s}\frac{M}{2\lambda L_0}$$

It can be checked quickly that this price level is consistent with $L = L_0$ and $C = 2\lambda L_0$.

(d) Plugging in the unchanged price level $P = \frac{1-s}{s}\frac{M}{\lambda L_0}$ and reformulating, we end up with

$$C = \frac{2s}{1+s}\lambda L_0 < \lambda L_0,$$
$$L = \frac{s}{1+s}L_0 < L_0$$

where the inequalities hold strictly since $s < 1$. Both consumption and employment fall, which results in involuntary unemployment. The lesson is that if productivity growth is not to result in either deflation or unemployment (or both), it must be accompanied by an expansion of demand which allows the economy to absorb the increase in potential output at an unchanged price level.

Problem 3 (10 Points)

(a) The loss function displays the central bank's aversion of variation in output gaps and inflation, where α is the relative weight the central bank puts on the output gap in comparison to inflation. The term $\delta \mathbb{E}_t L_{t+1}$ captures the future losses of the central bank recursively. Future deviations of output gap and inflation from their zero-targets enter today's losses discounted at rate δ. This is to say that the central bank minimizes the discounted expected stream of variations in output and inflation.

(b) Discretionary policy means that the central bank cannot commit to future values of the nominal interest rate, its instrument to conduct monetary policy. This implies that the central bank cannot commit to state-contingent plans for output gaps and inflation directly. Hence, it will act period by period minimizing x_t and π_t, taking expectations as given. This implies also, that $\delta \mathbb{E}_t L_{t+1}$ can be dropped from the optimization scheme.

(c) In a stationary long-run equilibrium, expected values correspond to their realizations. Hence, $\mathbb{E}_t x_{t+1} = x_{t+1} = x_t$ and $\mathbb{E}_t \pi_{t+1} = \pi_{t+1} = \pi_t$. Plugging these values in into equations (2) and (3), the system gets

$$\begin{aligned} \pi_t &= \beta \pi_t + \lambda x_t + u_t, \\ x_t &= x_t - \varphi \left(i_t - \pi_t - r_n \right). \end{aligned}$$

Reformulating implies

$$\begin{aligned} \pi_t &= \frac{\lambda}{1-\beta} x_t, \\ \pi_t &= i_t - r_n. \end{aligned}$$

By setting $i_t = r_n$, the central bank can achieve $\pi = 0$ and therefore $x = 0$, reaching the minimum possible loss.

(d) A negative cost push shock inflicts deflationary pressure on the economy. The central bank "leans against the wind" by decreasing the nominal interest rate. This stimulates output (IS-curve) and thereby reduces deflationary pressure (Phillips curve). Thereby, the central bank distributes the burden of the shock between π and x. As a result, the output gap jumps up and inflation deviates negatively on impact. When the shock fades out, so do the impulse responses of output gaps and inflation, so that x and π converge to long-run values asymptotically.