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Retake Exam
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ADVANCED MACROECONOMICS I

I. Short Questions (2 points each)

Mark the following statements as True (T) or False (F) and give a brief explanation of your answer in each case.

1.	In the Solow growth model, an acceleration of technological growth reduces aggregate savings on impact.
2.	For an elasticity of substitution between capital and labor larger than one, an increase in the capital-labor ratio is associated with an increase in the capital income share.
3.	A no-Ponzi-game condition is a necessary and sufficient condition for household optimality in the Ramsey-Cass-Koopmans model.
4.	The Lucas supply curve implies that unanticipated changes in monetary policy have an impact on output.
5.	According to the analysis of Clarida/Galí/Gertler, an increase in real interest rates reduces inflation.

Note: Write your answers on the blank paper provided to you.

II. 3 Problems (30 Points)

Problem 1 (10 Points)

Consider the Solow model without technological growth, where production obeys the Cobb-Douglas production function $Y = L^{1-\alpha}K^{\alpha}$ and where population grows at the exogenous rate n. The capital stock evolves according to $\dot{K} = sY - \delta K$.

(a) Derive the law of motion for the capital-labor ratio.

(b) Suppose now the economy experiences an increase in α.

(i) What happens to the income share of labor?

(ii) How does the growth path of the capital stock respond? Sketch the adjustment path using a semi-logarithmic scale.

(iii) Does the economy come closer to its golden-rule level of capital?

Problem 2 (10 Points)

Consider the following model of inflation and unemployment:

(1)	$\dot{\pi}(t) = -\beta u(t)$	Inflation dynamics
(2)	$u(t) = \sigma r(t)$	Unemployment determination
(3)	$r^T(t) = \phi[\pi(t) - \pi^T]$	Real interest rate target
(4)	$\dot{r}(t) = \rho[r^T(t) - r(t)]$	Real interest rate adjustment

where $\pi(t)$ is the inflation rate, $u(t)$ is the unemployment rate, and $r(t)$ the real interest rate. π^T is the central bank's inflation target. β, σ, ϕ and ρ are positive structural parameters.

(a) Explain the mechanism that gives rise to equation (2).

(b) Determine the stationary equilibrium of the model.

(c) Which decisions has the central bank to make in this model? Explain.

(d) Suppose the economy is initially in a disequilibrium with u and π below their respective equilibrium levels. Sketch the adjustment of u and π against the time axis.

Problem 3 (10 Points)

Consider the following version of the Clarida-Galí-Gertler (1999) model, in which the central bank solves the following problem (usual notation):

$$\min_{\{x_t,\pi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t \,, \quad L_t = \alpha x_t^2 + (\pi_t - \pi^T)^2$$
$$\text{s.t.} \quad \pi_t = \beta \mathbb{E}[\pi_{t+1}] + \lambda x_t + u_t$$
$$x_t = \mathbb{E}[x_{t+1}] - \varphi\,(i_t - \mathbb{E}[\pi_{t+1}]) + g_t$$

where u and g are white noise. α, β, λ and φ are given parameters. The inflation target π^T is exogenously given. $\mathbb{E}[y]$ are rational expectations with respect to y.

(a) Which variables are endogenous in this model?

(b) How does the objective function of the central bank simplify under discretionary policy?

(c) Assuming an optimal discretionary monetary policy, how do responses in x, π, and i differ *qualitatively* in presence of

(i) a positive shock to u?

(ii) a postive shock to g?

SOLUTION

Problem	Points				Total
I	1P T/F, 1P expl.				10
	(a)	(b)	(c)	(d)	
II1	4	6	-	-	10
II2	2	3	2	3	10
II3	3	3	4	-	10

I. Short Questions (10 Points)

1. **False.** Aggregate savings do not respond on impact and rises over time as technological progress accelerates.

2. **True.** An elasticity of substitution between capital and labor larger than one implies that any increase in the capital-labor ratio is absolutely larger than the concomitant change in the factor price ratio. Hence, the ratio of aggregate capital income to the aggregate wage bill must rise.

3. **False.** The no-Ponzi-game condition is necessary, but not sufficient.

4. **True.** Unanticipated changes in monetary policy induce price changes that are (mis)interpreted as relative price changes by agents and thus affect supply behavior.

5. **True.** For given expectations of future variables, an increase in real interest rates reduces the output gap according to the IS-curve, which works to reduce inflation via the Phillips curve.

II. 3 Problems (30 Points)

1. (a) Denote the capital-labor ratio as $k \equiv K/L$. The growth rate of k can be found by

$$\frac{\dot{k}}{k} = g_k = \frac{\partial \log \frac{K}{L}}{\partial t} = \frac{\partial \log K}{\partial t} - \frac{\partial \log L}{\partial t} = g_K - n$$

So that the law of motion is given by

$$\dot{k} = (g_K - n)k$$

where g_K can be computed from the definitions of Y and $\dot{K}$:

$$g_K = \frac{\dot{K}}{K} = \frac{sY - \delta K}{K} = s\left(\frac{L}{K}\right)^{1-\alpha} - \delta = sk^{\alpha-1} - \delta$$

so that

$$\dot{k} = sk^{\alpha} - (n + \delta)k$$

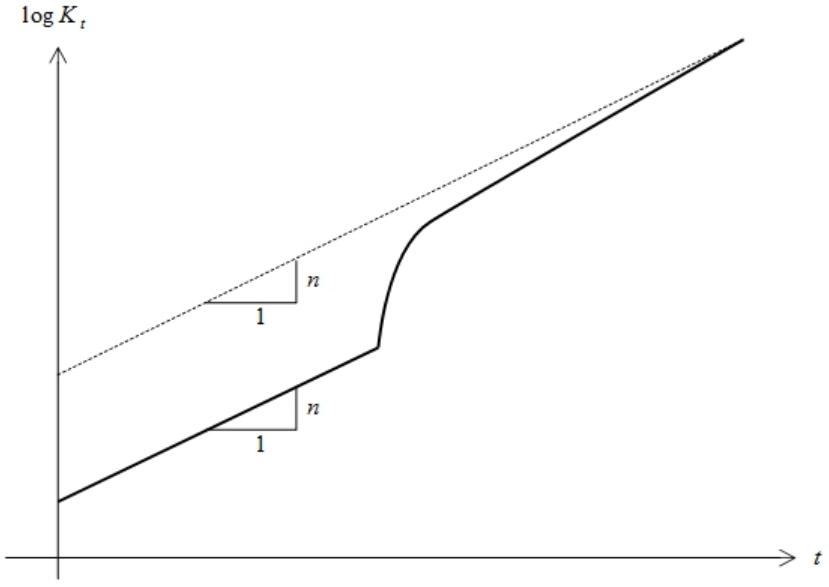
[4P]

(b) (i) For the Cobb-Douglas production function, the income share of labor $wL/Y = 1 - \alpha$, which follows from CRTS (to verify this, you can compute w explicitly from firms' optimality conditions). Hence, the income share of labor falls. [1.5P]

(ii) Note first that capital is a state variable and converges to its steady state level over time. The steady state level of capital per capita (k^*) increases, because it is increasing in α:

$$k^* = \left(\frac{s}{n + \delta}\right)^{\frac{1}{1-\alpha}}$$

For k to converge to its new steady state, g_k becomes positive so that growth of K accelerates transitorily:



[3P]

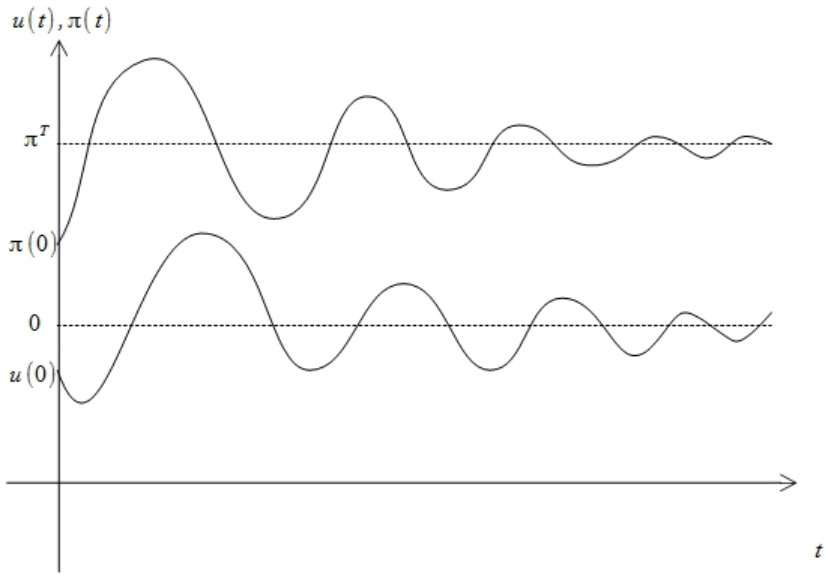
(iii) The golden rule level of capital obtains if $s = \alpha$. A rise in α widens the gap between the long-run level of capital and its golden rule level if the economy is dynamically efficient ($s < \alpha$), and narrows the gap if not ($s > \alpha$). [1.5P]

2. (a) Equ. (2) states that unemployment is increasing in the real interest rates. Higher interest rates reduce demand, and hence output and employment. Equation (2) reflects the joint operation of the IS-curve and Okun's Law. [2P]

(b) In a stationary equilibrium, $\dot{\pi} = 0$ and $\dot{r} = 0$, i.e. inflation and interest rates are constant. This implies $u = 0$ according to (1) and $r = r^T$ according to (4). $u = 0$ then implies $r = 0$ according to (2) so that $\pi = \pi^T$ follows from (3). [3P]

(c) The central banks sets the real interest rate as described by equations (3) and (4). This implies that a central bank must make a decision on the parameters ρ and ϕ (and π^T if the inflation target can be chosen freely). [2P]

(d) According to the (π-u) phase diagram, u and π take routes which let them oscillate around their long-run (stationary) equilibrium values.



Note that for below-equilibrium levels, unemployment first shrinks further while inflation rises. The oscillations get smaller over time for $\phi > 0$ and larger for $\phi < 0$. [3P]

3. (a) x_t, π_t, i_t, and their respective expectations are endogenously determined in the model. [3P]

   (b) Under discretionary monetary policy, the central bank cannot commit to future paths of x_t and π_t and therefore takes expectations of these variables as given. Instead of minimizing the discounted (expected) sum of losses, the central bank minimizes L_t in each period, i.e. the objective becomes

   $$\min_{x_t,\pi_t} L_t := \alpha x_t^2 + (\pi_t - \pi^T)^2 + F_t$$

   where F_t are (expected) future losses. [3P]

   (c) (i): In response to a cost-push shock u, optimal discretionary policy conducts a "leaning-against-the-wind" policy, i.e. spreads the burden of the shock between output gaps and inflation. In particular, the central bank absorbs part of the inflationary pressure by engineering an increase in nominal interest rates (which increases real interest rates) and thereby decreases output according to the IS curve. Hence, $i \uparrow$, $x \downarrow$ and $\pi \uparrow$.
   (ii): Operating through the IS curve, the central bank can completely offset demand shocks g by increasing the nominal interest rate with sufficient vigor so as to leave the output gap and inflation unchanged. In contrast to case (i), the shock can be neutralized in (ii) since demand shocks show up directly in the IS-curve, while cost push shocks appear in the Phillips curve. [4P]