ADVANCED MACROECONOMICS I

Final Exam, August 10, 2012

I. Short Questions (1 point each)

Mark the following statements as True (**T**) or False (**F**) on the **first page of the exam's sheets**. No explanations required in this part. One point for each correct answer. One point subtracted for each incorrect answer. Zero points for omitted questions.

1.	In the steady state of the Solow model, the capital-output ratio is constant.
2.	In the Ramsey-Cass-Koopmans model, the Euler equation expresses the intertemporal budget constraint of the representative household.
3.	In the U.S. and in Germany, the average share of gross investment in GDP is smaller than its contribution to the cyclical fluctuations of GDP.
4.	The RBC model explains employment fluctuations as a response of labor supply to variations in the real wage.
5.	In the Lucas Supply Function $y = b(p - E[p])$, the coefficient *b* is increasing in the variance of the aggregate demand shocks to which the economy is exposed.
6.	In new-keynesian macro models, an aggregate demand externality is a necessary and sufficient condition for the presence of nominal price rigidity.
7.	In the Fischer model of staggered price setting, a change in the money supply, announced one period ahead of its implementation, does not affect real output.
8.	In the Calvo model of staggered price setting, a change in the money supply, announced one period ahead of its implementation, does not affect real output.
9.	According to the Taylor Rule, both the nominal and the real interest rate should be increased in response to a rise in inflation.
10.	In the framework of Clarida/Galí/Gertler (JEL 1999), optimal monetary policy should not allow both inflation and output to exceed their target levels at the same time.

II. 4 Problems (30 points)

Problem 1 (6 points)

Consider Solow's model of economic growth with exogenous technical progress and zero population growth. What are the dynamic effects on per-capita consumption if there is a one-time increase in the number of workers at some point t_0?

Draw a diagram plotting per-capita consumption against time.

Problem 2 (8 points)

Consider a model along the lines of Clarida, Galí, and Gertler (JEL 1999), with the central bank pursuing an optimal discretionary monetary policy:

(1) $L = -\frac{1}{2}\left[\alpha x_t^2 + (\pi_t - \pi^*)^2\right]$

(2) $\pi_t = \lambda x_t + E_t \pi_{t+1} + u_t$

(3) $x_t = -\phi\left[i_t - E_t \pi_{t+1}\right] + E_t x_{t+1} + \overline{g}$

(4) $u_t = \rho u_{t-1} + \hat{u}_t;\ 0 < \rho < 1$

x: output gap; π: inflation rate; π^*: central bank's inflation target;
E: Rational expectations operator; *u*: cost-push disturbance; *i*: nominal interest rate; $\overline{g}$: exogenous autonomous demand (deterministic);
$\hat{u}_t$: random shock ($E_{t-1}\hat{u}_t = 0$)

a) Derive the first-order condition in x and π that must be satisfied by an optimal discretionary monetary policy.
b) Absent random shocks, what is the long-run equilibrium value of i_t?

Problem 3 (8 points)

Consider a Real Business Cycle model with technology A_t given by

$$A_t = \beta_0 + \beta_1 t + \beta_2 A_{t-1} + \varepsilon_t$$

t: time; ε_t: stochastic shock with mean zero (white-noise)

Consider the two polar cases $\beta_2 = 0$ and $\beta_2 = 1$. How do the responses of output, consumption, and labor supply to a shock $\varepsilon_t > 0$ differ between the two cases? (Assume $\varepsilon_{t+i} = 0,\ i = 1,2,...\infty$). Explain.

Problem 4 (8 points)

Consider the following rational-expectations model of an economy:

(1) $y_t = b(p_t - E_{t-1}[p_t])$	Lucas supply function
(2) $y_t = a(m_t - p_t) + \varepsilon_t^d$	Aggregate demand function
(3) $m_t = \overline{m} - cy_t$	Countercyclical monetary policy

where *y* denotes output, *p* the price level, *E* the expectations operator, and *m* the money supply. The disturbance ε^d is white noise and independently distributed with mean zero. The parameters *a*, *b* und *c* are constant and strictly positive.

Taking into account central bank behavior, solve the model for m_t, y_t and p_t as functions of $\overline{m}$ and ε_t^d.

ADVANCED MACROECONOMICS I

Abschlußklausur, 10. August 2012

I. Kurze Fragen (jeweils 1 Punkt)

Kennzeichnen Sie die folgenden Aussagen als Wahr (**W**) oder Falsch (**F**) auf dem **Deckblatt der Klausur**. Keine Erklärungen sind in diesem Teil verlangt. Ein Punkt bei korrekter Antwort. Ein Punkt Abzug für jede falsche Antwort. Null Punkte für ausgelassene Fragen.

1. Im stationären Gleichgewicht des Solow-Modells ist das Kapital-Output-Verhältnis konstant.
2. Im Ramsey-Cass-Koopmans-Modell drückt die Eulergleichung die intertemporale Budgetrestriktion des repräsentativen Haushalts aus.
3. In den U.S. und in Deutschland ist der durchschnittliche Anteil der Bruttoinvestitionen am BIP geringer als ihr Beitrag zu den zyklischen Schwankungen des BIP.
4. Das RBC Modell erklärt Beschäftigungsschwankungen als Reaktionen des Arbeitsangebots auf Änderungen des Reallohns.
5. In der Lucas-Angebotsfunktion $y = b(p - E[p])$ nimmt der Koeffizient *b* mit der Varianz der agggregierten Nachfrageschocks zu, denen die Volkswirtschaft ausgesetzt ist.
6. In neu-keynesianischen Modellen ist eine aggregierte Nachfrageexternalität eine notwendige und hinreichende Bedingung für das Vorhandensein einer nominalen Preisrigidität.
7. Im Fischer-Modell der gestaffelten Preissetzung hat eine Änderung des Geldangebots, die eine Periode vorher angekündigt wurde, keine Auswirkung auf den realen Output.
8. Im Calvo-Modell der gestaffelten Preissetzung hat eine Änderung des Geldangebots, die eine Periode vorher angekündigt wurde, keine Auswirkung auf den realen Output.
9. Laut der Taylor-Regel sollen sowohl der nominale als auch der reale Zinssatz erhöht werden, wenn die Inflation steigt.
10. Im Modell nach Clarida/Galí/Gertler (JEL 1999) sollte eine optimale Geldpolitik nicht zulassen, daß sowohl die Inflation als auch der Output ihre Zielniveaus zur gleichen Zeit überschreiten.

II. 4 Aufgaben (30 Punkte)

Problem 1 (6 Punkte)

Betrachten Sie Solows Wachstumsmodell mit exogenem technischen Fortschritt und einem Bevölkerungswachstum von Null. Wie sehen die dynamischen Effekte auf den Konsum pro Kopf aus, falls es einen einmaligen Anstieg in der Anzahl der Arbeiter zum Zeitpunkt t_0 gibt?

Zeichnen Sie ein Diagramm, das den Konsum pro Kopf gegen die Zeit abbildet.

Problem 2 (8 Punkte)

Betrachten Sie ein Model im Stile von Clarida, Galí und Gertler (JEL 1999), in dem die Zentralbank eine optimale diskretionäre Geldpolitik verfolgt:

(1) $L = -\frac{1}{2}\left[\alpha x_t^2 + (\pi_t - \pi^*)^2\right]$

(2) $\pi_t = \lambda x_t + E_t \pi_{t+1} + u_t$

(3) $x_t = -\phi\left[i_t - E_t \pi_{t+1}\right] + E_t x_{t+1} + \bar{g}$

(4) $u_t = \rho u_{t-1} + \hat{u}_t;\ 0 < \rho < 1$

x: Outputlücke; π: Inflationsrate; π^*: Zielinflationsrate der Zentralbank;
E: Rationaler Erwartungsoperator; *u*: Cost-push Schock; *i*: nominaler Zins;
$\bar{g}$: exogene autonome Nachfrage (deterministisch);
$\hat{u}_t$: Zufallsschock ($E_{t-1}\hat{u}_t = 0$)

a) Leiten Sie die Bedingung erster Ordnung für *x* und π her, die von einer optimalen diskretionären Geldpolitik erfüllt werden muß.
b) Welches ist in Abwesenheit von jeglichen Schocks das langfristige Gleichgewicht für *i*?

Problem 3 (8 Punkte)

Betrachten Sie ein Real Business Cycle Modell, in dem die Technologie A_t gegeben ist durch

$$A_t = \beta_0 + \beta_1 t + \beta_2 A_{t-1} + \varepsilon_t$$

t: Zeit; ε_t: stochastischer Schock mit Erwartungswert von Null (weißes Rauschen)
Betrachten Sie die zwei polaren Fälle $\beta_2 = 0$ und $\beta_2 = 1$. Wie unterscheiden sich in diesen Fällen die Auswirkungen eines Schocks $\varepsilon_t > 0$ auf Output, Konsum und das Arbeitsangebot? (Nehmen Sie an, daß $\varepsilon_{t+i} = 0,\ i = 1,2,...\infty$). Erklären Sie.

Problem 4 (8 Punkte)

Betrachten Sie das folgende Modell einer Volkswirtschaft mit rationalen Erwartungen:

(1) $y_t = b(p_t - E_{t-1}[p_t])$	Lucas Angebotsfunktion
(2) $y_t = a(m_t - p_t) + \varepsilon_t^d$	Aggregierte Nachfragefunktion
(3) $m_t = \overline{m} - cy_t$	Antizyklische Geldpolitik

Dabei bezeichnet *y* den Output, *p* das Preisniveau, *E* den Erwartungsoperator, und *m* das Geldangebot. Der Schock ist weißes Rauschen und unabhängig verteilt mit einem Erwartungswert von Null. Die Parameter *a, b* und *c* sind konstant und strikt positiv.

Lösen Sie das Modell unter Berücksichtigung des Zentralbankverhaltens nach m_t, y_t und p_t als Funktionen von $\overline{m}$ und ε_t^d .

ADVANCED MACROECONOMICS

Final Exam, August 10, 2012
Solution Outline

I. Short Questions (1 point each)

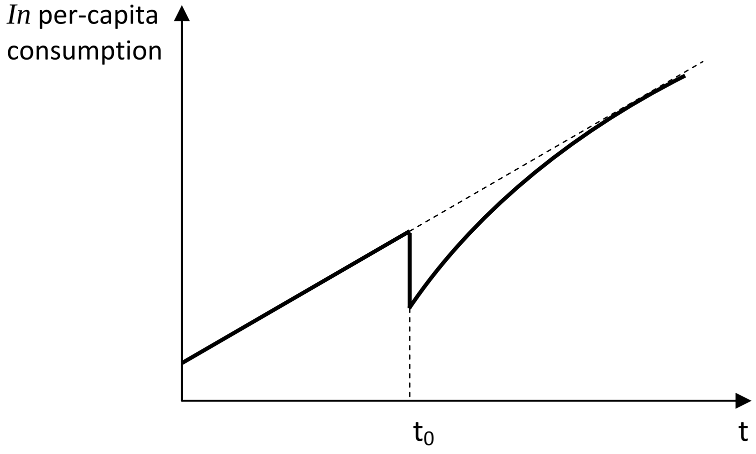
Mark the following statements as True (**T**) or False (**F**).

1. In the steady state of the Solow model, the capital-output ratio is constant.	**T**
2. In the Ramsey-Cass-Koopmans model, the Euler equation expresses the intertemporal budget constraint of the representative household.	**F**
3. In the U.S. and in Switzerland, the average share of gross investment in GDP is smaller than its contribution to the cyclical fluctuations of GDP.	**T**
4. The RBC model explains employment fluctuations as a response of labor supply to the variation in the real wage.	**T**
5. In the Lucas Supply Function $y = b\left(p - E\left[p\right]\right)$, the coefficient b is increasing in the variance of the aggregate demand shocks to which the economy is exposed.	**F**
6. In new-keynesian macro models, an aggregate demand externality is a necessary and sufficient condition for the presence of nominal price rigidity.	**F**
7. In the Fischer model of staggered price setting, a change in the money supply which is announced one period ahead of its implementation does not affect real output.	**F**
8. In the Calvo model of staggered price setting, a change in the money supply which is announced one period ahead of its implementation does not affect real output.	**F**
9. According to the Taylor Rule, both the nominal and the real interest rate should be increased in response to a rise in inflation.	**T**
10. In the framework of Clarida/Galí/Gertler (JEL 1999), optimal monetary policy should not allow both inflation and output to exceed their target levels at the same time.	**T**

II. 4 Problems (30 points)

Problem 1 (6 points)

A one-time increase in the labor supply at point t_0 amounts to an immediate fall in the capital-labor ratio of the economy. As a consequence, per-capita income and per-capita consumption are reduced on impact. But since the steady-state level of per-capita income is not affected at an unchanged saving rate, per-capita consumption gradually recovers and returns to its initial steady-state path.



Problem 2 (8 points)

a) The first order condition is the result of minimizing the loss function (1) subject to the Phillips curve (2). As monetary policy is discretionary the terms involving expectations are given exogenously. The problem can be solved either by setting up a Lagrangian function or by substituting (2) into the loss function (1) and then taking the derivative of the loss function with respect to x_t :

$$0 = \alpha\, x_t + (\pi_t - \pi^*)\lambda .$$

Rearranging results in the monetary policy rule (MPR): $x_t = -\frac{\lambda}{\alpha}(\pi_t - \pi^*)$.

b) From (2), in a stationary no-shock equilibrium, $\pi_t = E_t \pi_{t+1}$ implies $x_t = 0$. Given $x_t = 0$, the central bank chooses the optimal inflation rate $\pi_t = \pi_t^*$. Substituting these values into (3), we can solve for $i_t = \pi^* + \overline{g} / \phi$.

Problem 3 (8 points)

Answer follows Plosser (1989, p.56):

In the case $\beta_2 = 1$, the technology shock is permanent, i.e. it lifts A_t up to a permanently higher path and, therefore, does not create an incentive for intertemporal substitution. The income effect of the improved technology reduces labor supply if the intra-temporal substitution effect is not too strong. Output moves up to a permanently higher growth path and so does consumption. Thus, households use their permanently higher productivity to increase both their leisure and their consumption.

In the case $\beta_2 = 0$, the technology shock is strictly temporary so that technology is back to its initial equilibrium path in the next period. This means that the income effects on labor supply and on consumption are rather small. In contrast, there is a powerful intertemporal substitution effect, inducing households to work more while productivity is high. Thus, output experiences a short-lived boom due to both higher productivity and labor input. Consumption increases by much less than output as households wish to spread their extra consumption utility over time.

Summarizing the effects of $\varepsilon_t > 0$ on Y_t, C_t, and L_t:

	Y_t	C_t	L_t
$\beta_2 = 1$	up by less than $\partial Y_t / \partial \varepsilon_t$ if $L\downarrow$	up by a similar amount as Y_t	down (assuming income effect dominates in the absence of intertemporal substitution effect)
$\beta_2 = 0$	up by more than $\partial Y_t / \partial \varepsilon_t$ due to $L\uparrow$	Up by much less than Y_t (consumption smoothing)	up (two substitution effects, negligible income effect)

Problem 4 (8 points)

The first step is to solve the model for p_t as a function of $\overline{m}$, ε_t^d, and $E_{t-1}[p_t]$. Plugging (3) into (2) and eliminating y_t from (1) and (2) yields

$$p_t = \frac{(b+abc)E_{t-1}[p_t] + a\overline{m} + \varepsilon_t^d}{a+b+abc},$$

to which we apply the RE operator:

$$E_{t-1}[p_t] = E_{t-1}\left[\frac{(b+abc)E_{t-1}[p_t] + a\overline{m} + \varepsilon_t^d}{a+b+abc}\right] = \overline{m}$$

This rational expected price level can be used in (1) to solve (1)-(3) for y_t and p_t:

$$p_t = \overline{m} + \frac{1}{a+b+abc}\varepsilon_t^d \qquad\qquad y_t = \frac{b}{a+b+abc}\varepsilon_t^d$$