Macroeconomic Dynamics, 2014, Page 1 of 24. Printed in the United States of America.
doi:10.1017/S1365100513000783

FISCAL MULTIPLIERS IN A MONETARY UNION UNDER THE ZERO–LOWER–BOUND CONSTRAINT

STEFANIE FLOTHO
Albert-Ludwigs-Universität Freiburg

This paper analyzes government spending multipliers in a two-country model of a monetary union with price stickiness and home bias in consumption where monetary policy is constrained by the zero lower bound (ZLB) on the nominal interest rate. Government spending multipliers under this constraint are computed and compared with fiscal multipliers in normal times, that is, where the central bank sets the nominal interest rate via a Taylor rule. The trade elasticity and the parameter measuring home bias in consumption play an important role in determining the size of the multiplier. The multipliers are not necessarily large under the ZLB constraint. However, compared with the fiscal multipliers when the central bank sets the nominal interest rate according to a Taylor rule, the multipliers under the ZLB are bigger. Moreover, the persistence parameter of the binding ZLB plays a crucial role.

Keywords: Fiscal Multipliers, Monetary Union, Spillovers, Zero Lower Bound on Nominal Interest Rates

1. INTRODUCTION

During the past few decades of economic research and practice, fiscal policy has played a controversial role. Questions on whether fiscal policy should be used as a stabilization instrument garnered varying, sometimes opposite, answers. For a long time there has been a consensus that monetary policy should in general control the inflation rate and stabilize the business cycle (at least in a closed economy or in a small open economy with flexible exchange rates) and that activist fiscal policy, outside of so-called "automatic stabilizers" (such as unemployment insurance), should ensure fiscal sustainability with regard to deficits and debt [Kirsanova et al. (2009)].

I would like to thank participants in the annual meeting of the Verein für Socialpolitik 2012 in Göttingen, the annual meeting of the NOeG 2013 in Innsbruck, research seminars at the Universities of Freiburg and Kassel, and the 17th ICMAIF conference in Rethymno, Crete, 2013, and two anonymous referees for helpful comments that improved a previous version of this paper. Address correspondence to: Stefanie Flotho, Institut für allgemeine Wirtschaftsforschung, Abt. Wirtschaftstheorie, Albert-Ludwigs-Universität Freiburg, 79085 Freiburg, Germany; e-mail: stefanie.flotho@vwl.uni-freiburg.de.

 1365-1005/14

However, this view has changed during the past few decades for three main reasons. First, as the Japanese economy was hit by a severe recession during the 1990s with deflation and short-term interest rates near the zero lower bound (ZLB), monetary policy was useless in stabilizing the economy. Second, with the introduction of the European Monetary Union in 1999, the member states handed over their monetary policies to the central bank, which is in charge of dealing with disturbances on the aggregate. To stabilize country-specific shocks, member countries must rely on fiscal policy. Moreover, fiscal decisions on the decentralized level may influence monetary outcomes on the aggregate given that the two policies are interdependent. Third, as a result of the 2008–2009 Great Recession, there is an increasing role of fiscal policy worldwide. This includes macroeconomic research as well as practical implementation of government spending, taxation, and deficit financing as a stabilization tool to overcome the turmoil of the financial crisis, whereas monetary policy cannot act via the interest rate channel, as short-term policy rates are near zero in most countries.

This paper examines the role of fiscal policy in a dynamic stochastic general equilibrium (DSGE) two-country New Keynesian model of a monetary union that is enhanced with the additional assumption that the currency union is in a liquidity trap. The goal of the paper is to examine how effective fiscal policies are under the constraint that the central bank keeps the nominal interest rate at zero by analyzing fiscal multipliers in a currency union. Government spending multipliers under the ZLB constraint are computed and compared with fiscal multipliers in normal times; that is, the central bank can set the nominal interest rate via a Taylor rule.

The two countries are linked via different channels.[1] First, the trade channel is a direct linkage, usually assumed to have positive spillover effects via the net exports of the two countries. Second, the competitiveness channel is measured by an adjustment of the terms of trade, which matter because the nominal exchange rate between the two countries is fixed. Third, the indirect interest rate channel gives rise to negative spillover "beggar-thy-neighbors" effects. Hence, intraunion spillover plays an important role in determining the size of the multiplier. In this paper, an analytical solution for the multipliers is computed that shows dependence on the elasticity of trade and the share of imported to exported goods in the country. For standard calibration of the parameters, the multipliers are not necessarily large in both cases of the different monetary regimes. However, compared with the fiscal multipliers when the central bank sets the nominal interest rate according to a Taylor rule, the multipliers under the ZLB are bigger.

After the financial turmoil and the practical implementation of fiscal stimulus packages, research is trying to determine the effects of fiscal policy calculating fiscal multipliers. However, in the academic debate, there is no consensus in the size of the fiscal effect.[2] In standard New Keynesian models of a closed economy, government spending multipliers are usually below or near one at impact. Although some theoretical analysis on fiscal multipliers concludes that these are large in a liquidity trap [Eggertsson (2010); Christiano et al. (2011); Woodford (2011)],[3] other studies take a different view [Cogan et al. (2010)].[4]

Research on fiscal multipliers in a monetary union under a ZLB is still scarce. This paper adds to this strand of the literature. The main contribution of the paper is the explicit derivation of simple closed-form solutions for the government spending multipliers in the two-country New Keynesian model of a monetary union. The analytical solutions help to understand the transmission mechanism and the role of intraunion spillover.

Moreover, the results seem to contradict the literature, which suggests large multipliers under the ZLB constraint, but often these papers analyze closed economies or, in the case of open economies, flexible exchange rates. In this paper, the exchange rate between the two countries is fixed and spillovers to the neighbor country are different, as the exchange rate does not adjust. With a focus on intraunion trade spillovers, it complements and extends Cook and Devereux (2011), Farhi and Werning (2012), and Fujiwara and Ueda (2013).

The model in this paper is different from the one used in Farhi and Werning (2012). However, the conclusions are similar and complement the results by these authors. Fujiwara and Ueda (2013) is close to the analysis of this article. However, the authors consider an optimizing sticky price model of two countries that do not form a monetary union but set their monetary policies independent of each other. They compare analytical government spending multipliers in normal times with those that arise when both countries are caught in a liquidity trap. The multipliers are smaller when central banks can react to fiscal expansion by setting the nominal interest rate. In their model, it depends on the size of the intertemporal elasticity of substitution of consumption whether there are negative fiscal spillovers on the neighbor country. Using a similar setup, namely a standard new open economy macroeconomic model of two independent countries, Cook and Devereux (2011) compare government spending multipliers in the case where one or both countries are caught in the liquidity trap with the case when monetary policy operates under a Taylor rule. Fiscal policy expansions can help to stimulate the domestic economies, but there are significant negative cross-country spillovers via the terms of trade. Moreover, they find that the multipliers are even magnified in open economies in contrast with closed economies because in their setup, the exchange rate depreciates.[5]

Another strand of literature does not derive analytical closed-form solutions, but numerical values for the multipliers.[6] Simulating a large-scale DSGE model, the Euro Area and Global Economy Model, Gomes et al. (2010) obtain numerical values for fiscal policy multipliers in the ZLB. Their results suggest that multipliers are higher when the ZLB constraint binds than in normal times. The difference between multipliers in the two scenarios is higher for the United States than for the euro area. As a next step, they find that the fiscal stimulus package implemented by the United States even helps to overcome the periods with a binding ZLB, but the fiscal stimulus package of the euro area is not sufficient to end the ZLB constraint. Similar results are obtained by Coenen et al. (2012). In this meta-study of seven structural models, of which one is a two-region model and four are global models, the authors conclude that fiscal policy multipliers are enlarged when the nominal interest rates are constant. This result is valid across all the different models. Cwik and Wieland (2011) is another study comparing the effect of the fiscal stimulus packages of the euro area across several structural models. However, as indicated earlier, they conclude that the size of the multipliers does not increase under the ZLB. In contrast, Erceg and Lindé (2010) find very sizeable fiscal cross-country spillovers in a two-region model of a monetary union. They analyze the effects of asymmetric shocks in a currency union where the central bank usually sets the nominal interest rate via a Taylor rule, but is constrained by the ZLB for some periods. Fiscal policies in both regions (calibrated to the north and the south of the European Union) are independent. Governments can issue nominal debt to finance their deficits.

Section 2 introduces the theoretical model of the monetary union. Section 3 analyzes the spillover channels in this model, whereas Section 4 focuses on the role of monetary policy. In this section the case of a binding zero lower bound is compared with monetary policy in normal times. Section 5 gives the values of country-specific multipliers. The final section concludes.

2. A DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM TWO-COUNTRY MODEL OF A MONETARY UNION

The model used for the analysis is a DSGE model of two countries with a profound micro-foundation of optimization behavior combined with price stickiness to address problems of the open economy.[7] The interaction of monetary and fiscal policy in a monetary union is studied, to name a few, by Beetsma and Jensen (2004, 2005), Kirsanova et al. (2007), Galí and Monacelli (2008), and Ferrero (2009). The model in this paper closely follows Corsetti et al. (2010), who give a comprehensive and detailed description of a two-country DSGE model. Thus, the following exposition is kept as concise as possible. For details, the reader is referred to Corsetti et al. (2010). In the first section the assumptions of the model are described. Readers familiar with this model can skip to Section 2.2, where they will find the log-linearized version of the model.

2.1. The Monetary Union

The currency union consists of two countries, H(ome) and F(oreign). Total population is normalized to be equal to 1 in both countries and both countries have the same size. The preferences of households are symmetric across countries. A representative household in the home country chooses consumption C_t^H and work effort N_t^H to maximize lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^H\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^H\right)^{1+\eta}}{1+\eta}, \tag{1}$$

where E_0 denotes the expectation operator conditional upon the information set available at time 0. Households discount future utility at rate $\beta \in (0, 1)$. σ^{-1} is the intertemporal elasticity of substitution, and η denotes the inverse of the elasticity of labor supply.

The optimization is subject to a budget constraint of the household. One of the associated first-order conditions is the intertemporal consumption Euler equation,

$$\frac{U_C\left(C_t^H\right)}{P_t} = (1+i_t)\beta E_t \left[\frac{U_C\left(C_{t+1}^H\right)}{P_{t+1}}\right], \tag{2}$$

where P_t is a consumption price index (CPI) described later, and i_t is the yield paid on a domestic nominal bond.

The representative household decides the intratemporal allocation of the goods, produced either in the domestic country or abroad. Households have a home bias in consumption. So C_t^H is defined as a Dixit–Stiglitz consumption index,

$$C_t^H \equiv \left[\gamma^{\frac{1}{\phi}} \left(C_t^{HH}\right)^{\frac{\phi-1}{\phi}} + (1-\gamma)^{\frac{1}{\phi}} \left(C_t^{HF}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{3}$$

where the parameter $\phi > 0$ denotes the elasticity of substitution between domestic and foreign goods, i.e., the trade elasticity. The parameter γ is the weight of the consumption of the goods produced at home. If $\gamma > 1/2$, then home bias in consumption is modeled. The index C_t^{HH} resp. C_t^{HF} is an index of home consumption of domestic resp. imported goods and is defined over the continuum of differentiated varieties of goods produced in the home resp. the foreign country, given by

$$C_t^{HH} \equiv \left[\int_0^1 c_t^j(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}} \quad \text{resp.} \quad C_t^{HF} \equiv \left[\int_0^1 c_t^j(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ denotes the elasticity of substitution between varieties produced within a country.

The resulting demand function of the household in the home country for a good produced at home is given by

$$C_t^{HH} = \gamma \left[\frac{P_t(h)}{P_{H,t}}\right]^{-\theta} \left[\frac{P_{H,t}}{P_t}\right]^{-\phi} C_t^H,$$

where the consumption price index (CPI) is given by

$$P_t = \left(\gamma P_{H,t}^{1-\phi} + (1-\gamma) P_{F,t}^{1-\phi}\right)^{\frac{1}{1-\phi}},$$

with $P_{H,t}$ as the price index for home-produced goods and $P_{F,t}$ as the price index for foreign produced goods, both in domestic currency.

The fiscal authority in each country finances a stream of government expenditure G_t^H via lump-sum taxes such that government budgets are balanced each period. Government spending follows an AR(1)-process with coefficient ρ and is spent entirely on goods produced in the domestic country. Monetary policy is delegated to a common authority, the currency union's central bank. The instrument is a short-term interest rate. Usually, the central bank follows a Taylor rule.

The total demand Y_t^H for the goods produced in the home country consists of the demand by households living at home and the demand by households living in the foreign country. In addition, the home government expenditures are spent entirely on home goods.

As a result, total demand for domestically produced goods can be expressed as

$$Y_t^H = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\theta} \left\{\left(\frac{P_{H,t}}{P_t}\right)^{-\phi} \left[\gamma C_t^H + (1-\gamma) Q^{\phi} C_t^F\right] + G_t\right\}. \quad (4)$$

To derive this equation, it is assumed that the law of one price holds, i.e., $P_t(h) = e_t P_t^*(h)$. Here, e_t is the fixed nominal exchange rate (in the rest of the paper taken as the numeraire of the model). $Q_t = P_t^*/P_t$ denotes the real exchange rate. The price of the imports in terms of the exports, i.e., the terms of trade, for the home country is given by $T_t = P_{F,t}/P_{H,t}^*$.

Goods markets are characterized by monopolistic competition and nominal price rigidities. The assumption of Calvo (1983)-optimizing firms induces some degree of price stickiness in both countries. In each period, firms are able to set their prices with probability $(1-\alpha)$. So, with probability α, firms are not able to recalculate their prices. If firms can reset their prices, they are forward-looking and set prices optimally, taking into account the demand for domestic output. In this paper, the case of producer currency pricing (PCP) is explored: firms set the prices for the export goods in their domestic currency.[8] Prices of the exported good might vary when the nominal exchange rate fluctuates. However, in a currency union with a fixed exchange rate, this assumption simplifies even further. Moreover, as the elasticities of substitution of foreign and domestic goods are constant and assumed to be symmetric across countries, the law of one price holds for domestic and export markets of the producers: they set one optimal price. Thus, the terms of trade can be written as

$$T_t = \frac{P_{F,t}}{P_{H,t}}. \quad (5)$$

Financial markets are assumed to be complete at the national and international levels. This implies the following risk-sharing condition:[9]

$$\frac{U_C(C_{t+1}^H)P_t^H}{U_C(C_t^H)P_{t+1}^H} = \frac{U_C(C_{t+1}^F)P_t^F}{U_C(C_t^F)P_{t+1}^F}, \tag{6}$$

which states that markets are internationally efficient when the marginal utilities of consumption (weighted with the real exchange rate) are equal across countries.

2.2. Putting Everything Together: The Linearized Version of the Model

The equilibrium allocation is determined by the aggregate demand equation (4), the consumption Euler equation (2), and the pricing equations of the supply side for both the home and the foreign country. Moreover, the equilibrium can be characterized by the efficient market condition (6) and the evolution of the terms of trade (5). A log-linearized version of the equations around a symmetric steady state with zero inflation yields a set of equations that are quite tractable. In the linearized model, lower case letters denote log deviations from the steady state. The economic conditions in the domestic country can be summarized by the following three equations:

$$\pi_t^H = \beta E_t \pi_{t+1}^H + k\left[(1-\gamma)q_t + \sigma c_t^H + \eta y_t^H\right], \tag{7}$$

$$y_t^H = c_y\left[2\phi\gamma(1-\gamma)q_t + \gamma c_t^H + (1-\gamma)c_t^F\right] + (1-c_y)g_t^H, \tag{8}$$

$$c_t^H = E_t c_{t+1}^H - \frac{1}{\sigma}\left\{\bar{\imath}_t - E_t\left[\gamma\pi_t^H + (1-\gamma)\pi_t^F\right]\right\}, \tag{9}$$

with $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\eta\theta)$, where α is the probability of the Calvo-price adjustment model, i.e., the fraction of firms that cannot reset their prices each period.

The first equation, (7), is the log-linearized version of the aggregate supply equation of the domestic economy, the New Keynesian Philips curve (NKPC). The producer inflation rate in the home country, π_t^H, depends on the discounted value of the expected inflation rate of the next period, thus containing the forward-looking behavior of a NKPC because firms cannot adjust their prices every period. β is the discount rate. Moreover, the domestic inflation rate positively depends on the domestic output y_t^H and domestic consumption c_t^H. The terms of trade q_t have a positive effect on domestic inflation. A rise in the terms of trade q_t results in higher domestic output, more work effort, and rising prices.

The second equation, (8), is the log-linearized aggregate demand relation (4) of the economy. Domestic output is either consumed by the domestic government, g_t^H, with a share of $(1-c_y)$, where $c_y = C/Y$ is the steady-state consumption share of output or is determined by domestic consumption, γc_t^H, or exported to the foreign country. As domestic goods are traded, relative prices between the two countries determine the choice of home output. Higher terms of trade q_t switch demand toward goods produced in the home country. The increase in domestic government expenditures is like a demand shock in the economy. Higher spending in the home country raises demand for the goods sold in the home country. As a consequence, the monopolistically competitive firms in the home country increase their demand for labor, which results in higher real wages and higher marginal costs. Those firms that can set new prices according to the Calvo contracts increase their prices, leading to higher inflation. In sum, a positive domestic demand shock in the form of government spending leads to higher domestic inflation rates and higher domestic product regardless of the design of fiscal or monetary policy. Via the intraunion trade channel, the increase in domestic government spending has an effect on the foreign economy and then again on the home country. Depending on the size of the spillover effects, either positive or negative cross-country multipliers can be observed, which is discussed in detail in Section 3.

The third equation, (9), is the log-linearized Euler equation, (2), using the explicit assumption of the utility function and an approximation of CPI inflation. Here, $\bar{\imath}$ denotes the short-term interest rate, the monetary policy instrument that is given for the home country. The interest rate channel is an indirect channel for spillover. However, changes in the short-term interest rate (because of a reaction of the monetary policy instrument to an increase in aggregate output or inflation) might lead to negative spillover on the foreign country and give rise to so-called beggar-thy-neighbor effects.

Analogous equations hold for the Foreign economy:

$$\pi_t^F = \beta E_t \pi_{t+1}^F + k\left[-(1-\gamma)q_t + \sigma c_t^F + \eta y_t^F\right], \quad (10)$$

$$y_t^F = c_y\left[-2\phi\gamma(1-\gamma)q_t + \gamma c_t^F + (1-\gamma)c_t^H\right] + (1-c_y)g_t^F, \quad (11)$$

$$c_t^F = E_t c_{t+1}^F - \frac{1}{\sigma}\left\{\bar{\imath}_t - E_t\left[\gamma\pi_t^F + (1-\gamma)\pi_t^H\right]\right\}. \quad (12)$$

Inflation dynamics in the Foreign country are given by (10). The terms of trade have the opposite effect on the inflation rate, as an increase in the terms of trade, as mentioned previously, results in a shift of demand toward Home-produced goods which implies less work effort in the Foreign country and, hence, decreasing prices. The analogous counterpart of the impact of a change in the terms of trade can be found in the aggregate demand relationship (11) for the foreign country, whose output depends negatively on the terms of trade. The counterpart of the domestic Euler equation is given by equation (12).

The model is completed by a log-linear approximation of the terms of trade,

$$q_t = q_{t-1} + \pi_t^F - \pi_t^H, \quad (13)$$

and the log-linearized approximation of the international risk-sharing condition (6),

$$\sigma\left(c_t^F - c_t^H\right) = (1-2\gamma)q_t. \quad (14)$$

In the subsequent analysis, the following notation for a generic variable x is useful. By $x^W \equiv \frac{1}{2}(x^H + x^F)$, the *aggregate level* of a variable, i.e., a weighted average of the Home and the Foreign variables, is denoted, whereas $x^R \equiv x^F - x^H$ denotes the *relative level*. Using this notation, a variable for the Home, respectively for the Foreign country can be expressed as

$$x^H = x^W - \frac{1}{2}x^R, \quad \text{resp.} \quad x^F = x^W + \frac{1}{2}x^R. \tag{15}$$

The symmetric structure of the log-linearized equations provides a tractable system of the model. By taking aggregates and relatives of the equations, the system falls into a set of equations describing the behavior of the aggregate variables and a set describing relative variables. The two sets are disjoint, a fact that helps to derive analytical solutions.

The aggregate part of the monetary union is described by the following three equations:

$$\begin{aligned}
\pi_t^W &= \beta E_t \pi_{t+1}^W + k\left(\sigma c_t^W + \eta y_t^W\right), \\
y_t^W &= c_y c_t^W + (1 - c_y) g_t^W, \\
c_t^W &= E_t c_{t+1}^W - \frac{1}{\sigma}\left(\bar{\iota}_t - E_t \pi_{t+1}^W\right).
\end{aligned}$$

The relative part of the monetary union is given by

$$\begin{aligned}
\pi_t^R &= \beta E_t \pi_{t+1}^R - 2k(1-\gamma)q_t + k\sigma\left(c_t^R\right) + k\eta y_t^R, \\
y_t^R &= c_y\left[-4\phi\gamma(1-\gamma)q_t + (2\gamma - 1)(c_t^R)\right] + (1 - c_y) g_t^R, \\
c_t^R &= E_t c_{t+1}^R + \frac{1}{\sigma}(2\gamma - 1) E_t \pi_{t+1}^R.
\end{aligned}$$

Note that the system of the relative variables does not depend on monetary policy. The solution to this system can be derived independent of the short-term interest rate, as this cancels out when the difference of the consumption Euler equations is taken.[10] Together with the log-linearized approximation of the international risk-sharing condition, (14), this system of equations can be written as two equations. Substituting relative demand and the risk-sharing condition into the Phillips curve and the relative Euler equation yields a system of rational difference equations with two endogenous forward-looking variables:

$$\begin{aligned}
\pi_t^R &= \beta E_t \pi_{t+1}^R + \Phi q_t + \Psi g_t^R, \\
q_t &= E_t q_{t+1} - E_t \pi_{t+1},
\end{aligned}$$

where $\Phi = -k\{1 + \eta c_y[4\phi\gamma(1-\gamma) + (2\gamma - 1)^2\sigma^{-1}]\}$ and $\Psi = k\eta(1 - c_y)$. The system has a determinate rational-expectations equilibrium if the following parameter restriction holds: $0 < \Phi < 2(1+\beta)/\beta$. Appendix A.1 discusses the determinacy condition of the system and derives the solution by the method of undetermined coefficients. The solutions for relative output and relative consumption are given by

$$c_t^R = \frac{-(1-2\gamma)\sigma^{-1}k\eta(1-c_y)\rho}{(1-\beta\rho)(1-\rho)-\rho k\{1+\eta c_y[4\phi\gamma(1-\gamma)+(2\gamma-1)^2\sigma^{-1}]\}}\, g_t^R, \tag{16}$$

$$\begin{aligned} y_t^R &= c_y[-4\phi\gamma(1-\gamma)-(1-2\gamma)^2\sigma^{-1}]q_t + (1-c_y)g_t^R \\ &= \frac{(1-c_y)[(1-\beta\rho)(1-\rho)-k\rho]}{(1-\beta\rho)(1-\rho)-\rho k\{1+\eta c_y[4\phi\gamma(1-\gamma)+(2\gamma-1)^2\sigma^{-1}]\}}\, g_t^R. \end{aligned} \tag{17}$$

Several conclusions can be drawn from these results. In the case of symmetric government expenditures across countries, i.e., $g_t^H = g_t^F$, relative government spending is zero. In this case trade linkages do not play a role. However, when the government expenditures are asymmetric across countries, i.e., $g_t^H = -g_t^F$, the relative variables depend on relative government spending. In that case, the response of the relative variables to a change in relative government spending is determined by the parameters indicating the spillover channels.

3. THE ROLE OF SPILLOVER EFFECTS

To explore the mechanism by which asymmetric government expenditures influence both countries, it is assumed that foreign government spending is higher than home government expenditures. Thus g_t^R is positive. Besides the direct positive effect on relative demand captured by $(1-c_y)$ in (17), the government expenditures have a negative effect through the terms of trade captured by $-4\phi\gamma(1-\gamma)$ and the efficient market condition $(2\gamma-1)$. The impact through the latter channel is absent when the share of imports and the share of exports are equal and thus $\gamma = 1/2$, and the channel is positive as long as the home bias parameter $\gamma > 1/2$. The expenditure-switching effect of the terms of trade dominates the risk-sharing channel; i.e., the multiplier on relative output is negative for the following value restriction of the parameters: $4(1-\gamma)\gamma\phi + (1-2\gamma)^2\sigma^{-1} > 0$. This can be rewritten as $(1-\phi\sigma)4\gamma(\gamma-1)+1 > 0$. The combination of the trade elasticity, the share of imports, and the coefficient of risk aversion plays a crucial role. As in Corsetti et al. (2010), what role the factor $(1-\phi\sigma)$ plays is analyzed. First, assume that $(1-\phi\sigma) = 0$. Then the condition is true for all values of home bias γ, which can be seen in Figure 1, which depicts the relative output multiplier depending on γ for various values of ϕ.[11] In this case the multiplier is constant for all values of γ. Second, if $(1-\phi\sigma) < 0$, then the restriction holds irrespective of the home bias. Hence, in this case the multiplier is negative. When the condition holds, goods across countries are substitutes. Higher government

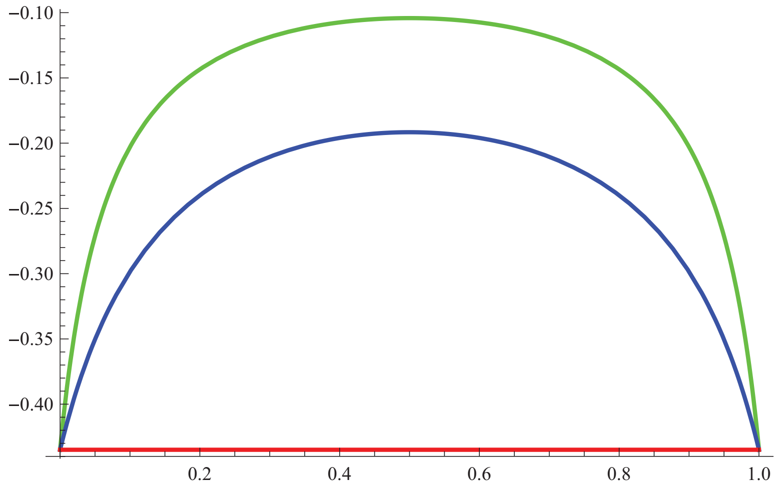

FIGURE 1. Graphical representation of the relative output multiplier depending on γ for values of $\phi = 0.5$ (bottom), $\phi = 0.7$ (middle), and $\phi = 1$ (top).

spending in the foreign country leads via an increase of the foreign producer price inflation rate to an increase in the terms of trade. Expenditures switches toward goods produced in the home country, and demand for the goods produced in the foreign countries falls. In this case, an increase in foreign government spending leads to a positive spillover for the home country. Third, if $(1-\phi\sigma) > 0$ (goods are complements), then the parameter restriction holds for values of $\gamma < \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\phi\sigma}{1-\phi\sigma}}$ and $\gamma > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\phi\sigma}{1-\phi\sigma}}$; i.e., home bias is high. The goods across countries are complements and an increase in g_t^F leads to an increase in demand for goods in both countries. So the comovement of output in both countries is positive. However, when home bias gets too large, then the expenditure-switching effect prevails. Figure 1 clearly shows that the effect of an increase in the trade elasticity on the relative government spending multiplier is positive. The higher the substitutability between goods produced across countries, the smaller the expenditure-switching effect.

The relative government spending multiplier of relative consumption is positive if $\gamma < 1/2$ and negative for $\gamma > 1/2$, as can be seen in Figure 2. With home bias an increase in foreign government spending increases the relative price of the foreign goods. The risk-sharing condition (14) implies that with higher terms of trade, home consumption must increase relative to foreign consumption. Thus, foreign government spending crowds out foreign consumption via the risk-sharing channel because of efficient markets. This effect is bigger the higher the home bias parameter is. An increase in the intertemporal elasticity of substitution and the intratemporal elasticity of substitution between home and foreign goods dampens

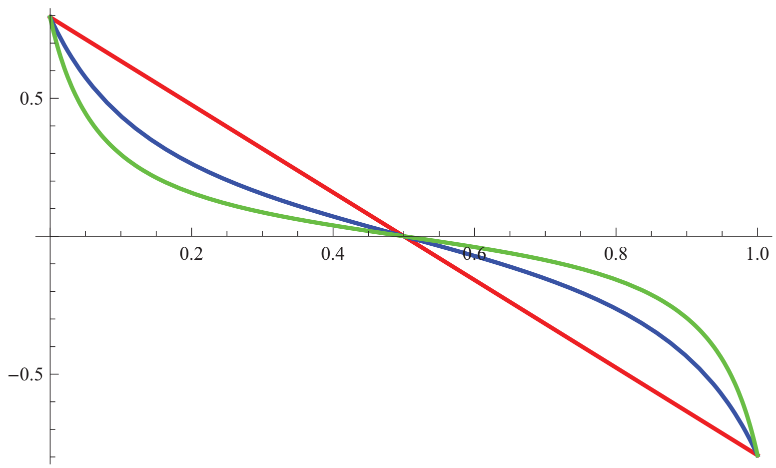

FIGURE 2. Graphical representation of the relative consumption multipliers depending on the home bias for various $\phi = 0.5$ (straight line), $\phi = 0.7$ (middle), and $\phi = 1$ (bottom until intersection with axis).

the effect, which is depicted in Figure 2 for various values of ϕ. However, the effect is absent if the shares of imports and exports are equal.

So far, just the relative part of the economy has been discussed. However, to gain insight into the effects of government expenditures on country-specific output and consumption, the impact on the aggregate variables of the monetary union must be considered. This is done in the following section.

4. THE ROLE OF MONETARY POLICY

In contrast to the relative part of the monetary union, spillover via trade channels does not play a role in the aggregate part of the economy. However, the aggregate equations depend on the nominal interest rate, which determines consumption in both countries via the Euler equation. How the solution of the aggregate variables differ across regimes of monetary policy is explored in more detail in this section. First, I assume that the central bank can freely adjust the nominal interest rate; that is, multipliers in normal times are derived in Section 4.1. In this case the central bank follows a Taylor rule. These results are compared with the case where the central bank is constrained by the ZLB on the nominal interest rate (Section 4.2).

4.1. The Multiplier with a Taylor Rule

In this section, the central bank follows a Taylor rule when setting the nominal interest rate. To simplify computations, the Taylor rule is given by

$$\bar{\imath} = \phi_\pi \pi_t^W + \phi_y y_t^W,$$

where the coefficients $\phi_y > 0$ and $\phi_\pi > 1$ to follow the Taylor principle, and the target inflation rate is set to zero. Inserting the monetary policy rule into the aggregate consumption equation and solving the system of the resulting equation and the aggregate Phillips curve (see Appendix A.2 for details on determinacy and the solution method) yields the following multipliers:

$$\frac{dy_t^W}{dg_t^W} = \frac{(-1+c_y)[-(-1+c_y)\phi_y(-1+\beta\rho)-(1+k\phi_\pi)\sigma+\rho(1+\beta+k-\beta\rho)\sigma]}{c_y[\phi_y(1-\beta\rho)+\eta k(\phi_\pi-\rho)]+[k(\phi_\pi-\rho)+(1-\rho)(1-\beta\rho)]\sigma},$$

$$\frac{dc_t^W}{dg_t^W} = \frac{-(1-c_y)[\phi_y(1-\beta\rho)+\eta k(\phi_\pi-\rho)]}{c_y[\phi_y(1-\beta\rho)+\eta k(\phi_\pi-\rho)]+[k(\phi_\pi-\rho)+(1-\rho)(1-\beta\rho)]\sigma}.$$

When government spending in both countries is asymmetric, then in the aggregate, government expenditures are zero and do not have an effect on aggregate inflation and aggregate consumption. However, when expenditures are not asymmetric, then aggregate government spending has an effect on the monetary union. The aggregate consumption multiplier is less than zero. An increase in aggregate government spending creates inflationary pressure on the monetary union, besides the direct effect of an increase in aggregate output. The central bank raises the nominal interest rate, which dampens consumption in the aggregate and for both countries via the Euler equation. This is an example of the indirect interest rate channel in the monetary union. An increase of government spending in any of the countries has an effect on the short term interest rate in the two-country model which lead to negative spillover "beggar-thy-neighbors" effects on the other country.

4.2. The Multiplier with a Binding Zero Lower Bound

In modeling a binding zero lower bound constraint, this paper follows Eggertsson (2010) and Woodford (2011). The shock leading to a binding zero lower bound on the nominal interest rate is modeled as a spread Δ_t in the nominal interest rate.

A central bank setting the nominal interest rate according to

$$\bar{\iota} = \max\left[0, \bar{r}_t + \phi_\pi \pi_t^W + \phi_y y_t^W\right], \quad \textbf{(18)}$$

where $\bar{r}_t = -\Delta_t$ sets the nominal interest rate according to the Taylor rule as long as $\bar{r}_t \geq 0$. However, if the increase in the spread is high enough so that $\bar{r}_t \equiv r_L < 0$ is small enough to make the zero lower bound on $\bar{\iota}$ binding, the nominal interest rate is set to zero. Further assume that with a probability of $0 < \mu < 1$, the financial disturbance (i.e., higher credit spreads) still prevails in the next period, and thus the real rate is still at the low level. On returning to the normal level [with probability $(1-\mu)$]—i.e. higher credit spreads decrease, such that the nominal interest rate can take positive values—the economy recovers from the financial disturbance and returns to the normal level for the economic variables.

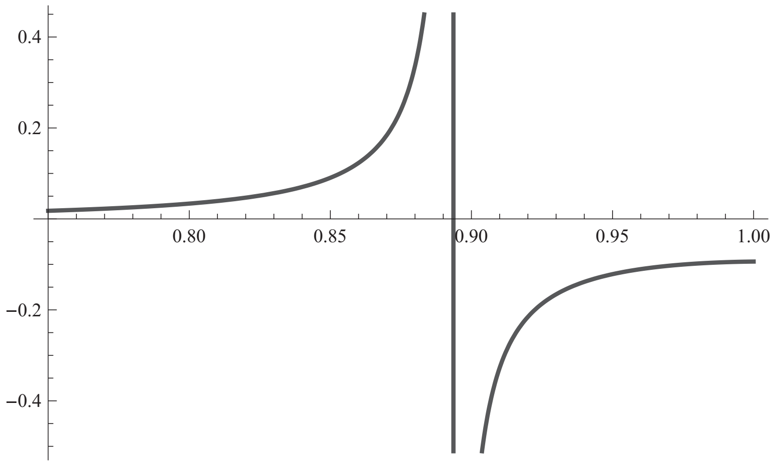

FIGURE 3. Graphical representation of the aggregate consumption multiplier under the ZLB depending on μ.

Concerning government expenditures, it is assumed that government purchases are increased during the time of distress to g_L^H, g_L^F, but once the economy recovers, i.e., the zero lower bound does not bind any longer, government spending in both countries is reduced to zero (which is the long-run equilibrium value). Thus, as in the previously mentioned papers, this paper analyzes government spending multipliers under a binding ZLB.

The government spending multipliers for aggregate output and aggregate consumption while the ZLB binds are given by

$$\frac{dc_L^W}{dg_L^W} = \frac{(1 - c_y)\eta k \mu}{(1 - \beta\mu)(1 - \mu)\sigma - k(\sigma + \eta c_y)\mu}, \tag{19}$$

$$\frac{dy_L^W}{dg_L^W} = \frac{(1 - c_y)[(1 - \beta\mu)\sigma(1 - \mu) - \mu k \sigma]}{(1 - \beta\mu)(1 - \mu)\sigma - k(\sigma + \eta c_y)\mu}. \tag{20}$$

However, these solutions are determinate under the following parameter restriction: $(\sigma - \mu)(1 - \beta\mu) - \mu\sigma(c_y \eta k + k\sigma) > 0$. For a discussion of the determinacy conditions and the derivation of the complete solution the reader is referred to Appendix A.3.

Two observations are noteworthy. First, it is obvious that both multipliers crucially depend on the parameter μ. This parameter determines whether the multipliers are negative or positive, as can be seen in Figure 3, which depicts as an example the aggregate consumption multiplier depending on the parameter μ. Up to a value $\bar{\mu}$ the consumption multiplier is positive and then becomes negative $\bar{\mu}$ depends on the parameters of the model (see Appendix A.4).

Second, the aggregate output multiplier is not necessarily greater than 1. This is in contrast to the results of studies that examine the multiplier for closed economies [Christiano et al. (2011); Woodford (2011)]. With a binding ZLB, the relative inflation rate is lower than in normal times. As the nominal exchange rate is fixed, the terms of trade dampen the expansionary effect of government expenditures.

To complete the analysis, the relative part of the economy has to be explored. A discussion of determinacy and the complete solutions can be found in Appendix A.3. The effects of relative government spending on relative output and relative consumption are given for the following parameter restriction, which has to hold:

$$(1+\beta\mu) < -k\{1+\eta c_y[4\phi\gamma(1-\gamma)+(2\gamma-1)^2\sigma^{-1}]\} < \left(1+\frac{2}{\mu}+\frac{1}{\beta\mu}\right).$$

Then the relative consumption multiplier and the relative output multiplier are given by

$$\frac{dc_L^R}{dg_L^R} = \frac{-(1-2\gamma)\sigma^{-1}k\eta(1-c_y)\mu}{(1-\beta\mu)(1-\mu)-\mu k\{1+\eta c_y[4\phi\gamma(1-\gamma)+(2\gamma-1)^2\sigma^{-1}]\}}, \tag{21}$$

$$\frac{dy_L^R}{dg_L^R} = \frac{(1-c_y)[(1-\beta\mu)(1-\mu)-k\mu]}{(1-\beta\mu)(1-\mu)-\mu k\{1+\eta c_y[4\phi\gamma(1-\gamma)+(2\gamma-1)^2\sigma^{-1}]\}}. \tag{22}$$

The parameters of trade, ϕ and γ, determine the size of the multiplier, as in the case of the trade spillover for the relative part of the monetary union, discussed in Section 3. However, the parameter μ again plays a crucial role here, and parameter restrictions for μ for a determinate rational solution to exist have to hold (see the Appendix).

5. COUNTRY-SPECIFIC MULTIPLIERS

How are country-specific multipliers to be determined? So far, solutions for aggregate and relative variables have been derived. For any variable, the solutions can be written as $x_t^W = ag_t^W$ for the aggregate part and $x_t^R = bg_t^R$ for the relative part, with constants $a, b \in \mathbf{R}$. Substituting these relations into $x^H = x^W - \frac{1}{2}x^R$, the effect of domestic government spending g^H on the home variable x^H and the effect of foreign government spending g^F on the home variable are given by

$$\frac{dx^H}{dg^H} = \frac{a+b}{2} \quad \text{and} \quad \frac{dx^H}{dg^F} = \frac{a-b}{2}.$$

TABLE 1. Calibration

Parameter		Value
Discount factor	β	0.99
Risk aversion	σ	2
Weight on government expenditures	$(1 - c_y)$	0.2
Trade elasticity	ϕ	1
Share of home-traded goods	γ	0.8
Probability of keeping prices	α	0.75
Elasticity of substitution between home-traded goods	θ	6
Inverse of the Frisch labor supply elasticity	η	1.5
Taylor coefficient on aggregate inflation	ϕ_π	1.5
Taylor coefficient on aggregate output	ϕ_y	0.5/4
AR term of government spending	ρ	0.9
Probability of a binding ZLB next period	μ	0.8

The case for the foreign country yields

$$\frac{dx^F}{dg^H} = \frac{a-b}{2} \quad \text{and} \quad \frac{dx^F}{dg^F} = \frac{a+b}{2}.$$

Hence, cross-country multipliers are symmetric.

To get an idea of the magnitude of the multipliers in the model used in this paper, the parameters are calibrated as in Corsetti et al. (2010). A period is a quarter, so the discount factor β is set to 0.99. The share of government spending is assumed to be one-fifth. This implies a value of $c_y = 0.8$. Calvo contracts can be adjusted with a probability of $1 - \alpha = 0.25$. Hence, prices are adjusted on the average once a year. Concerning the household's preferences, the Frisch labor elasticity is set to $\eta = 1.5$ and the coefficient of risk aversion is equal to $\sigma = 2$. The share of imports corresponds to 10%. Hence $\gamma = 0.9$. The elasticity of substitution between home and foreign goods (i.e., the trade elasticity) is set to $\phi = 1$, whereas the elasticity of substitution of home-traded goods is $\theta = 6$. For the coefficients of the Taylor rule, the values $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$ are standard [Eggertsson (2010)]. Government expenditures are persistent, with a parameter $\rho = 0.9$. For the probability μ that the ZLB is binding in the next period, the value $\mu = 0.8$ is used, as in Cook and Devereux (2011). The value of 0.903, as suggested by Eggertsson (2010) or Woodford (2011), lies in the critical range of values for this model. Table 1 summarizes the calibration of the model.

The values for the aggregate and the relative multipliers, both for the case of normal monetary policy and for the case of a binding zero bound on the interest rate, are given in Table 2, whereas Table 3 shows the values for country-specific multipliers. So an increase of aggregate government spending by EUR 1 increases aggregate output under the Taylor rule by EUR 0.13, and crowds out aggregate consumption by EUR 0.08. In contrast, under the ZLB, the interest rate channel

TABLE 2. Numerical multipliers for the baseline calibration for aggregate and relative variables

Variable	Taylor	Rule	Zero	Lower B
	dg_t^W	dg_t^R	dg_t^W	dg_t^R
dy_t^W	0.134187		0.226916	
dy_t^R		−0.143457		0.248303
dc_t^W	−0.0822657		0.0336455	
dc_t^R		−0.157069		0.0220899

TABLE 3. Numerical multipliers for the baseline calibration for home and foreign variables

Variable	Taylor	Rule	Zero	Lower B
	dg_t^H	dg_t^F	dg_t^H	dg_t^F
dy_t^H	−0.00463499	0.138822	0.23761	−0.0106935
dy_t^F	0.138822	−0.00463499	−0.0106935	0.23761
dc_t^H	−0.119667	0.0374016	0.0278677	0.00577776
dc_t^F	0.0374016	−0.119667	0.00577776	0.0278677

is closed, leading to an increase of aggregate consumption by EUR 0.03 and an increase of EUR 0.22 of aggregate output.

The multiplier with a binding ZLB is higher than in normal times, as the central bank does not increase the nominal interest rate, which dampens the expansive effect of government expenditures.

6. CONCLUSION

Using a two-country New Keynesian model of a monetary union government, spending multipliers for both the home and the foreign country and the monetary union as a whole are computed when monetary policy follows a Taylor rule. These are compared with the values of the multipliers when the nominal interest rate is zero. The size of the various multipliers depends on the combination of the intraunion competitiveness parameters and is not necessarily large when the ZLB constraint binds. The parameter measuring the persistence of the binding ZLB constraint plays a crucial role.

This paper shows that studies on fiscal multipliers that predict large government spending multipliers do not apply to the monetary union. Another set of models have to be considered. The terms of trade under a fixed exchange rate dampen the expansive effects of an increase in government spending. The paper is normative. However, to see how the model matches data on the European Union is left for future research.

To keep the model set-up tractable to get closed-form solutions, many simplifying assumptions have been made in this setup. However, whether the results are robust when LCP is introduced is left as a next issue to examine. Adding different country sizes would be preferred as well.

One of the key missing elements is an endogenous time length of the ZLB constraint. In this paper, the focus is on analyzing the problem of how governments should interact while the central bank keeps the nominal interest rate at zero. How fiscal policy can react to terminate the ZLB constraint and stabilize the economy is an open question.

Moreover, as the first fiscal stimulus packages have been phased out and the economy has started to recover, the question turns to fiscal consolidation packages. How can deficits be avoided so that the economy does not turn into a recession again? Thus, debt dynamics should be included in the analysis.

NOTES

1. As the monetary union is modeled to be a closed economy, external spillover effects with a third country outside the union are not considered.
2. This paper focuses on a theoretical analysis of the issue. For examples of the empirical analysis of fiscal multipliers, see the articles by Blanchard and Perotti (2002), Mountford and Uhlig (2009), Romer and Romer (2010), Traum and Yang (2011), and Ilzetzki et al. (2013). All articles find estimates of government spending multipliers close to one on impact. However, depending on the data set and identification scheme, values are estimated in a broader range and can even become negative.
3. This group defend their analytical result with the argument that fiscal policy is more effective when the nominal interest is bounded at zero. When the central bank cannot counteract the fiscal expansion and increasing inflation rates by increasing nominal interest rates, they do not crowd out consumption or investment.
4. These authors, for example, use a model database of different macro models to investigate the effects of the American Recovery and Reinvestment Plan, and conclude that the stimulus is much smaller than predicted by Romer and Bernstein (2009) when analyzed in New Keynesian type models. In a companion article, Cwik and Wieland (2011) perform the same analysis with regard to the European fiscal stimulus packages: Their results suggest smaller multipliers.
5. This is in contrast with open economy results that imply that exchange rates appreciate under a fiscal expansion and thus multipliers are dampened by a fall in net exports.
6. The list of papers discussed here is not complete.
7. Thus it belongs to the growing class of New Open Economy models, which started with the seminal paper by Obstfeld and Rogoff (1995). Monetary policy interaction is studied by Clarida et al. (2002) and Benigno (2004), among others.
8. The assumption of PCP makes the model more tractable. To study the question at hand with local currency pricing is left for future research.
9. This assumption goes back to Backus and Smith (1993). For more details see Corsetti et al. (2008). Again this assumption is imposed for tractability of the model. However, it is interesting to explore how the results of this paper change when financial markets are incomplete. This is left for future analysis.
10. This is not true when the two countries do not form a currency union, but have independent monetary policies setting different nominal interest rates. To explore this issue in this model setup and compare with results as found in the literature [Cook and Devereux (2011)] is left for future research.
11. The graph is given for $\sigma = 2$. For the calibration of the other variables see Table 1.

REFERENCES

Backus, D.K. and G.W. Smith (1993) Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics* 35(3–4), 297–316.

Beetsma, R.M. and H. Jensen (2004) Mark-up fluctuations and fiscal policy stabilization in a monetary union. In Special Issue. *Journal of Macroeconomics* 26(2), 357–376.

Beetsma, R.M. and H. Jensen (2005) Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics* 67, 320–352.

Benigno, P. (2004) Optimal monetary policy in a currency area. *Journal of International Economics* 63, 293–320.

Blanchard, O. and R. Perotti (2002) An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *Quarterly Journal of Economics* 117(4), 1329–1368.

Calvo, G. (1983) Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12(3), 383–398.

Christiano, L., M. Eichenbaum, and S. Rebelo (2011) When is the government spending multiplier large? *Journal of Political Economy* 119(1), 78–121.

Clarida, R., J. Gali, and M. Gertler (2002) A simple framework for international monetary policy analysis. *Journal of Monetary Economics* 49(5), 879–904.

Coenen, G., C.J. Erceg, C. Freedman, D. Furceri, M. Kumhof, R. Lalonde, D. Laxton, J. Lindé, A. Mourougane, D. Muir, S. Mursula, C. de Resende, J. Roberts, W. Roeger, S. Snudden, M. Trabandt, and J. in't Veld (2012) Effects of fiscal stimulus in structural models. *American Economic Journal: Macroeconomics* 4(1), 22–68.

Cogan, J.F., T. Cwik, J.B. Taylor, and V. Wieland (2010) New Keynesian versus old Keynesian government spending multipliers. *Journal of Economic Dynamics and Control* 34(3), 281–295.

Cook, D. and M.B. Devereux (2011) Optimal fiscal policy in a world liquidity trap. *European Economic Review* 55(4), 443–462.

Corsetti, G., L. Dedola, and S. Leduc (2008) International risk sharing and the transmission of productivity shocks. *Review of Economic Studies* 75(2), 443–473.

Corsetti, G., L. Dedola, and S. Leduc (2010) Optimal monetary policy in open economies. In B.M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics*, Volume 3 of *Handbook of Monetary Economics*, Chapter 16, pp. 861–933. Amsterdam: Elsevier.

Cwik, T. and V. Wieland (2011) Keynesian government spending multipliers and spillovers in the euro area. *Economic Policy* 26(67), 493–549.

Eggertsson, G.B. (2010) What fiscal policy is effective at zero interest rates? In *NBER Macroconomics Annual 2010,* Vol. 25, NBER Chapters, pp. 59–112. National Bureau of Economic Research.

Erceg, C.J. and J. Lindé (2010) Asymmetric shocks in a currency union with monetary and fiscal handcuffs. In *NBER International Seminar on Macroeconomics 2010*, NBER Chapters, pp. 95–135. National Bureau of Economic Research.

Farhi, E. and I. Werning (2012) Fiscal Multipliers: Liquidity Traps and Currency Unions. Working paper 18381, National Bureau of Economic Research.

Ferrero, A. (2009) Fiscal and monetary rules for a currency union. *Journal of International Economics* 77(1), 1–10.

Fujiwara, I. and K. Ueda (2013) The fiscal multiplier and spillover in a global liquidity trap. *Journal of Economic Dynamics and Control* 37(7), 1264–1283.

Galí, J. and T. Monacelli (2008) Optimal monetary and fiscal policy in a currency union. *Journal of International Economics* 76(1), 116–132.

Gomes, S., P. Jacquinot, R. Mestre, and J. Sousa (2010) Global Policy at the Zero Lower Bound in a Large-Scale DSGE Model. Working paper series 1254, European Central Bank.

Ilzetzki, E., E.G. Mendoza, and C.A. Végh (2013) How big (small?) are fiscal multipliers? *Journal of Monetary Economics* 60(2), 239–254.

Kirsanova, T., C. Leith, and S. Wren-Lewis (2009) Monetary and fiscal policy interaction: The current consensus assignment in the light of recent developments. *Economic Journal* 119(541), F482–F496.

Kirsanova, T., M. Satchi, D. Vines, and S. Wren-Lewis (2007) Optimal fiscal policy rules in a monetary union. *Journal of Money, Credit and Banking* 39(7), 1759–1784.

Mountford, A. and H. Uhlig (2009) What are the effects of fiscal policy shocks? *Journal of Applied Econometrics* 24(6), 960–992.

Obstfeld, M. and K. Rogoff (1995) Exchange rate dynamics redux. *Journal of Political Economy* 103(3), 624–60.

Romer, C. and J. Bernstein (2009) *The Job Impact of the American Recovery and Reinvestment Plan*. Office of the President-Elect.

Romer, C.D. and D.H. Romer (2010) The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks. *American Economic Review* 100(3), 763–801.

Traum, N. and S.-C.S. Yang (2011) Monetary and fiscal policy interactions in the post-war U.S. In Special Issue on Monetary and Fiscal Interactions in Times of Economic Stress, *European Economic Review* 55(1), 140–164.

Woodford, M. (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

Woodford, M. (2011) Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics* 3(1), 1–35.

APPENDIX: THE RATIONAL EXPECTATIONS SOLUTIONS

According to Woodford (2003, Appendix C), a linear model of two difference equations with rational expectations with two nonpredetermined endogeneous variables of the form

$$E_t X_{t+1} = AX_t + BG_t$$

has a determinate equilibrium if and only if the matrix A has both eigenvalues bigger than 1 in modulus. This is equivalent to the following conditions: either, in the first case,

$$\det A > 1, \quad \det A - \operatorname{tr} A > -1, \quad \text{and} \quad \det A + \operatorname{tr} A > -1,$$

or, in the second case,

$$\det A - \operatorname{tr} A < -1 \quad \text{and} \quad \det A + \operatorname{tr} A < -1.$$

In what follows, these conditions determine the parameter restrictions of the model.

The model is solved by the method of undetermined coefficients. A solution is conjectured and substituted into the equation. Under the assumption that government expenditures follow an AR(1)-process $G_{t+1} = \rho G_t + \varepsilon_t$ with a parameter $0 < \rho < 1$ and i.i.d. zero-mean disturbances ε_t, the solution simplifies and the unknown coefficients can be determined. In general, assume that $G_t = g_t$ is a 1×1 variable, $X = (x1, x2)^T$. Conjecture a solution of the form $x1 = ag_t$ and $x2 = bg_t$. The coefficients a, b are to be determined. Substituting this conjecture into the preceding system implies a linear system in two equations and two unknowns, which has to be solved.

A.1. THE RELATIVE PART OF THE MONETARY UNION

The equations for the relative part are given as follows:

$$\pi_t^R = \beta E_t \pi_{t+1}^R + \Phi q_t + \Psi g_t^R,$$
$$q_t = E_t q_{t+1} - E_t \pi_{t+1}.$$

Writing this in matrix form as previously with $X_t = (\pi_t^R, q_t)^T$ and $G_t = g_t^R$ yields the following matrices of coefficients, where $\Phi = -k\{1 + \eta c_y[4\phi\gamma(1-\gamma) + (2\gamma-1)^2\sigma^{-1}]\}$ and $\Psi = k\eta(1 - c_y)$:

$$A = \frac{1}{\beta}\begin{pmatrix} 1 & -\Phi \\ 1 & \beta - \Phi \end{pmatrix}, \qquad B = \frac{1}{\beta}\begin{pmatrix} \Psi \\ \Psi \end{pmatrix}.$$

As the determinant of the system is $\det A = 1$, the system yields determinacy according to the preceding conditions if and only if the following restrictions for the parameters hold:

$$(1 - \beta) < \Phi < (3\beta + 1).$$

Using the method of undetermined coefficients as described previously yields the solution

$$a = \frac{\Psi(1-\rho)}{(1-\beta\rho)(1-\rho) + \Phi\rho},$$
$$b = \frac{-\Psi\rho}{(1-\beta\rho)(1-\rho) + \Phi\rho},$$

which implies that

$$\pi_t^R = \frac{k\eta(1-c_y)(1-\rho)}{(1-\beta\rho)(1-\rho) - \rho k\{1 + \eta c_y[4\phi\gamma(1-\gamma) + (2\gamma-1)^2\sigma^{-1}]\}} g_t^R, \quad \textbf{(A.1)}$$
$$q_t = \frac{-k\eta(1-c_y)\rho}{(1-\beta\rho)(1-\rho) - \rho k\{1 + \eta c_y[4\phi\gamma(1-\gamma) + (2\gamma-1)^2\sigma^{-1}]\}} g_t^R. \quad \textbf{(A.2)}$$

Together with the risk-sharing condition, relative consumption can be determined.

A.2. THE AGGREGATE PART OF THE MONETARY UNION WITH A TAYLOR RULE

Writing this in matrix form as previously with $X_t = (\pi_t^W, c_t^W)^T$ and $G_t = g_t^W$ yields the following matrix of coefficients:

$$A = \frac{\sigma}{\beta}\begin{pmatrix} \frac{1}{\sigma} & -\frac{1}{\sigma}(c_y\eta + k\sigma) \\ -1 + \frac{\beta\phi_\pi}{\sigma} & \beta(1 + \frac{\phi_y c_y}{\sigma}) + (c_y\eta + k\sigma) \end{pmatrix}.$$

The solution of the system is determinate for certain parameter restriction.

Using the method of undetermined coefficients as described previously yields the solution

$$a = \frac{(-1+c_y)k[\phi_y + \eta(-1+\rho)]\sigma}{c_y[\phi_y + \eta k(\phi_\pi - \rho) - \beta\phi_y\rho] + [k(\phi_\pi - \rho) + (-1+\rho)(-1+\beta\rho)]\sigma},$$

$$b = \frac{(-1+c_y)[\phi_y + \eta k(\phi_\pi - \rho) - \beta\phi_y\rho]}{c_y[\phi_y + \eta k(\phi_\pi - \rho) - \beta\phi_y\rho] + [k(\phi_\pi - \rho) + (-1+\rho)(-1+\beta\rho)]\sigma}.$$

A.3. THE ZERO-LOWER-BOUND SOLUTION

To analyze the parameter restrictions for a determinate solution with a binding zero lower bound, the procedure in Eggertsson (2010) is applied. Assuming that, with a probability of μ, the financial distress holds in the next period, the system of aggregate variables can be written in the form

$$E_t X_{t+1} = AX_t + BG_t,$$

where the matrix A is given by

$$A = \frac{1}{\beta\mu}\begin{pmatrix} 1 & (c_y\eta k + k\sigma) \\ -\sigma & \sigma\beta + \sigma(c_y\eta k + k\sigma) \end{pmatrix}.$$

The determinant is $\det A = \sigma/\beta\mu^2 > 1$. As the trace of the matrix is given by $\operatorname{tr} A = \frac{1+\beta\sigma+k\sigma(c_y\eta+\sigma)}{\beta\mu}$, the determinacy condition $\det A + \operatorname{tr} A > -1$ always holds, whereas $\det A - \operatorname{tr} A > -1$ holds if and only if $(\sigma - \mu)(1-\beta\mu) - \mu\sigma(c_y\eta k + k\sigma) > 0$.

Exploring the relative part of the economy, the following results can be derived. The system can be simplified to two equations for $X_t = (\pi_t^R, q_t)$:

$$E_t X_{t+1} = AX_t + BG_t,$$

where the matrix A is given by

$$A = \frac{1}{\beta\mu}\begin{pmatrix} 1 & -\Phi \\ 1 & \beta - \Phi \end{pmatrix}.$$

The determinant is $\det A = \frac{1}{\mu} > 1$. The two conditions implying determinacy hold if and only if the following restriction holds: $1 + \beta\mu < \Phi < (1 + 2/\mu + 1/\beta\mu)$.

Here $\Phi = -k\{1 + \eta c_y[4\phi\gamma(1-\gamma) + (2\gamma - 1)^2\sigma^{-1}]\}$.

Under a binding ZLB, the system of equations can be rewritten taking into account that with a probability of μ the economy is still in a recession the next period:

$$\begin{aligned} \pi_L^W &= \beta\mu\pi_L^W + k\left(\sigma c_L^W + \eta y_L^W\right), \\ y_L^W &= c_y c_L^W + (1-c_y)g_L^W, \\ c_L^W &= \mu c_L^W + \sigma^{-1}(\mu\pi^W + r_L). \end{aligned}$$

This is a system of three equations in three unknowns with the exogenous policy variables g_L^W and r_L.

A solution is easily derived while the ZLB is still binding. The system of aggregate equations can be written in the form $AX = BG$, where $X = (y_L^W, c_L^W, \pi^W)^T$ is the vector of endogenous aggregate variables in the depressed economy and $G = (g_L^W, r_L)^T$ is the vector of policy instruments. A and B are the appropriate matrices of the coefficients. Applying the inverse of A on both sides yields the solution $X = \Omega G$, where Ω is given by

$$\Omega = \begin{pmatrix} \frac{(-1+c_y)\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma}{c_y\eta k\mu+\left[-1+(1+\beta+k)\mu-\beta\mu^2\right]\sigma} & \frac{c_y(-1+\beta\mu)}{c_y\eta k\mu+\left[-1+(1+\beta+k)\mu-\beta\mu^2\right]\sigma} \\ \frac{(-1+c_y)\eta k\mu}{c_y\eta k\mu+\left[-1+(1+\beta+k)\mu-\beta\mu^2\right]\sigma} & \frac{1-\beta\mu}{-c_y\eta k\mu+\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma} \\ -\frac{(-1+c_y)\eta k(-1+\mu)\sigma}{c_y\eta k\mu+\left[-1+(1+\beta+k)\mu-\beta\mu^2\right]\sigma} & \frac{k(c_y\eta+\sigma)}{-c_y\eta k\mu+\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma} \end{pmatrix}.$$

The equations for the relative part of the economy are given by

$$\begin{aligned} \pi_L^R &= \beta\mu\pi_L^R - 2k(1-\gamma)q_L + k\left(\sigma c_L^R + \eta y_L^R\right), \\ y_L^R &= c_y\left[-4\phi\gamma(1-\gamma)q_L + (2\gamma-1)c_L^R\right] + (1-c_y)g_L^R, \\ c_L^R &= \mu c_L^R + \sigma^{-1}(2\gamma-1)\mu\pi_L^R, \\ \sigma c_L^R &= (1-2\gamma)q_L. \end{aligned}$$

The system of the relative part of the economy is given by $A^R X = B^R G$, where $X^R = (y_L^R, c_L^R, \pi^R, q_L)^T$ is the vector of endogenous aggregate variables in the depressed economy and $G = g_L^R$ is the policy instrument. A^R and B^R are the appropriate matrices of the coefficients. Applying the inverse of A^R on both sides yields the solution $X = \Omega^R G$, where Ω^R is given by

$$\begin{pmatrix} -\frac{(-1+c_y)\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma}{\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma+c_y\eta k\mu\left[-1+\gamma(4-4\phi\sigma)+4\gamma^2(-1+\phi\sigma)\right]} \\ -\frac{(-1+c_y)\eta(-1+2\gamma)k\mu}{\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma+c_y\eta k\mu\left[-1+\gamma(4-4\phi\sigma)+4\gamma^2(-1+\phi\sigma)\right]} \\ \frac{(-1+c_y)\eta k(-1+\mu)\sigma}{\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma+c_y\eta k\mu\left[-1+\gamma(4-4\phi\sigma)+4\gamma^2(-1+\phi\sigma)\right]} \\ \frac{(-1+c_y)\eta k\mu\sigma}{\left[1-(1+\beta+k)\mu+\beta\mu^2\right]\sigma+c_y\eta k\mu\left[-1+\gamma(4-4\phi\sigma)+4\gamma^2(-1+\phi\sigma)\right]} \end{pmatrix}.$$

A.4. PARAMETER RESTRICTIONS

For the aggregate consumption multiplier to be positive, the parameter μ must be less than a threshold value $\bar{\mu}$; i.e., $\mu < \bar{\mu}$, where

$$\bar{\mu} = \frac{c_y\eta k + \sigma(1+\beta+k) - \beta\sqrt{\frac{c_y^2\eta^2k^2+2c_y\eta k(1+\beta+k)\sigma+[\beta^2+2\beta(-1+k)+(1+k)^2]\sigma^2}{\beta^2\sigma^2}}}{2\beta\sigma}.$$

The relative output multiplier is zero for

$$\mu = \frac{1 + \beta + k - \sqrt{1 - 2\beta + \beta^2 + 2k + 2\beta k + k^2}}{2\beta}$$

or

$$\mu = \frac{1 + \beta + k + \sqrt{1 - 2\beta + \beta^2 + 2k + 2\beta k + k^2}}{2\beta}.$$